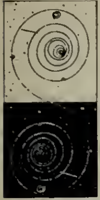

The Project Physics Course

Reader 1

Concepts of Motion

The Project Physics Course

Reader

UNIT 1 Concepts of Motion

A Component of the
Project Physics Course

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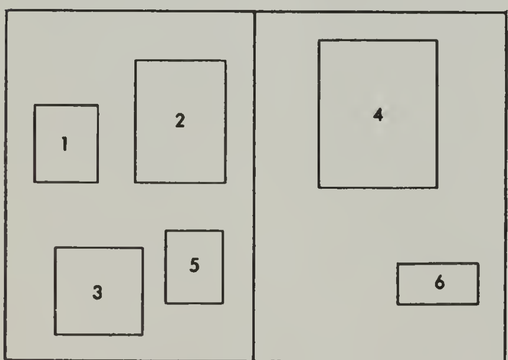

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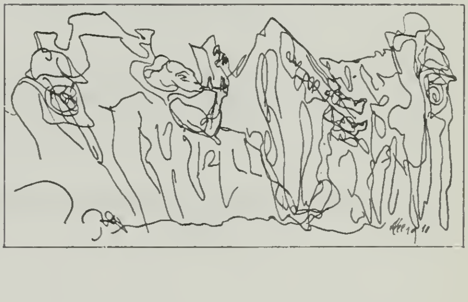
This is not a physics textbook. Rather, it is a physics reader, a collection of some of the best articles and book passages on physics. A few are on historic events in science, others contain some particularly memorable description of what physicists do; still others deal with philosophy of science, or with the impact of scientific thought on the imagination of the artist.

There are old and new classics, and also some little-known publications; many have been suggested for inclusion because some teacher or physicist remembered an article with particular fondness. The majority of articles is not drawn from scientific papers of historic importance themselves, because material from many of these is readily available, either as quotations in the Project Physics text or in special collections.



This collection is meant for your browsing. If you follow your own reading interests, chances are good that you will find here many pages that convey the joy these authors have in their work and the excitement of their ideas. If you want to follow up on interesting excerpts, the source list at the end of the reader will guide you for further reading.

Reader 1

Table of Contents

A still photo of the Calder mobile shown in motion
on the cover.

An outstanding contemporary theoretical physicist reminisces informally about science and its role in society. Feynman stresses the importance in science, and elsewhere, of admitting that one does not know all the answers.

1 The Value of Science

Richard P. Feynman

An excerpt from *Frontiers of Science*, 1958.

From time to time, people suggest to me that scientists ought to give more consideration to social problems—especially that they should be more responsible in considering the impact of science upon society. This same suggestion must be made to many other scientists, and it seems to be generally believed that if the scientists would only look at these very difficult social problems and not spend so much time fooling with the less vital scientific ones, great success would come of it.

It seems to me that we do think about these problems from time to time, but we don't put full-time effort into them—the reason being that we know we don't have any magic formula for solving problems, that social problems are very much harder than scientific ones, and that we usually don't get anywhere when we do think about them.

I believe that a scientist looking at nonscientific problems is just as dumb as the next guy—and when he talks about a nonscientific matter, he will sound as naive as anyone untrained in the matter. Since the question of the value of science is not a scientific subject, this discussion is dedicated to proving my point—by example.

The first way in which science is of value is familiar to everyone. It is that scientific knowledge enables us to do all kinds of things and to make all kinds of things. Of course if we make good things, it is not only to the credit of science; it is also to the credit of the moral choice which led us to good work. Scientific knowledge is an enabling power to do either good or bad—but it does not carry instructions on how to use it. Such power has evident value—even though the power may be negated by what one does.

I learned a way of expressing this common human problem on a trip to Honolulu. In a Buddhist temple there, the man in charge explained a little bit about the Buddhist religion for tourists, and then ended his talk by telling them he had something to say to them that they would *never* forget—and I have never forgotten it. It was a proverb of the Buddhist religion:

"To every man is given the key to the gates of heaven; the same key opens the gates of hell."

What then, is the value of the key to heaven? It is true that if we lack clear instructions that determine which is the gate to heaven and which the gate to hell, the key may be a dangerous object to use, but it obviously has value. How can we enter heaven without it?

The instructions, also, would be of no value without the key. So it is evident that, in spite of the fact that science could produce enormous horror in the world, it is of value because it *can* produce *something.*

Another value of science is the fun called intellectual enjoyment which some people get from reading and learning and thinking about it, and which others get from working in it. This is a very real and important point and one which is not considered enough by those who tell us it is our social responsibility to reflect on the impact of science on society.

Is this mere personal enjoyment of value to society as a whole? No! But it is also a responsibility to consider the value of society itself. Is it, in the last analysis, to arrange things so that people can enjoy things? If so, the enjoyment of science is as important as anything else.

But I would like *not* to underestimate the value of the world view which is the result of scientific effort. We have been led to imagine all sorts of things infinitely more marvelous than the imaginings of poets and dreamers of the past. It shows that the imagination of nature is far, far greater than the imagination of man. For instance, how much more remarkable it is for us all to be stuck—half of us upside down—by a mysterious attraction, to a spinning ball that has been swinging in space for billions of years, than to be carried on the back of an elephant supported on a tortoise swimming in a bottomless sea.

I have thought about these things so many times alone that I hope you will excuse me if I remind you of some thoughts that I am sure you have all had—or this type of thought—which no one could ever have had in the past, because people then didn't have the information we have about the world today.

For instance, I stand at the seashore, alone, and start to think. There are the rushing waves . . . mountains of molecules, each stupidly minding its own business . . . trillions apart . . . yet forming white surf in unison.

Ages on ages . . . before any eyes could see . . . year after year . . . thunderously pounding the shore as now. For whom, for what? . . . on a dead planet, with no life to entertain.

Never at rest . . . tortured by energy . . . wasted prodigiously by the sun . . . poured into space. A mite makes the sea roar.

Deep in the sea, all molecules repeat the patterns of one another till complex new ones are formed. They make others like themselves . . . and a new dance starts.

Growing in size and complexity . . . living things, masses of atoms, DNA, protein . . . dancing a pattern ever more intricate.

Out of the cradle onto the dry land . . . here it is standing . . . atoms with consciousness . . . matter with curiosity.

Stands at the sea . . . wonders at wondering . . . I . . . a universe of atoms . . . an atom in the universe.

THE GRAND ADVENTURE

The same thrill, the same awe and mystery, come again and again when we look at any problem deeply enough. With more knowledge comes deeper, more wonderful mystery, luring one on to penetrate deeper still. Never concerned that the answer may prove disappointing, but with pleasure and confidence we turn over each new stone to find unimagined strangeness leading on to more wonderful questions and mysteries—certainly a grand adventure!

It is true that few unscientific people have this particular type of religious experience. Our poets do not write about it; our artists do not try to portray this remarkable thing. I don't know why. Is nobody inspired by our present picture of the universe? The value of science remains unsung by singers, so you are reduced to hearing—not a song or a poem, but an evening lecture about it. This is not yet a scientific age.

Perhaps one of the reasons is that you have to know how to read the music. For instance, the scientific article says, perhaps, something like this: "The radioactive phosphorus content of the cerebrum of the rat decreases to one-half in a period of two weeks." Now, what does that mean?

It means that phosphorus that is in the brain of a rat (and also in mine, and yours) is not the same phosphorus as it was two weeks ago, but that all of the atoms that are in the brain are being replaced, and the ones that were there before have gone away.

So what is this mind, what are these atoms with consciousness? Last week's potatoes! That is what now can *remember* what was going on in my mind a year ago—a mind which has long ago been replaced.

That is what it means when one discovers how long it takes for the atoms of the brain to be replaced by other atoms, to note that the thing which I call my individuality is only a pattern or dance. The atoms come into my brain, dance a dance, then go out; always new atoms but always doing the same dance, remembering what the dance was yesterday.

THE REMARKABLE IDEA

When we read about this in the newspaper, it says, "The scientist says that this discovery may have importance in the cure of cancer." The paper is only interested in the use of the idea, not the idea itself. Hardly anyone can understand the

importance of an idea, it is so remarkable. Except that, possibly, some children catch on. And when a child catches on to an idea like that, we have a scientist. These ideas do filter down (in spite of all the conversation about TV replacing thinking), and lots of kids get the spirit—and when they have the spirit you have a scientist. It's too late for them to get the spirit when they are in our universities, so we must attempt to explain these ideas to children.

I would now like to turn to a third value that science has. It is a little more indirect, but not much. The scientist has a lot of experience with ignorance and doubt and uncertainty, and this experience is of very great importance, I think. When a scientist doesn't know the answer to a problem, he is ignorant. When he has a hunch as to what the result is, he is uncertain. And when he is pretty darn sure of what the result is going to be, he is in some doubt. We have found it of paramount importance that in order to progress we must recognize the ignorance and leave room for doubt. Scientific knowledge is a body of statements of varying degrees of certainty—some most unsure, some nearly sure, none *absolutely* certain.

Now, we scientists are used to this, and we take it for granted that it is perfectly consistent to be unsure—that it is possible to live and *not* know. But I don't know whether everyone realizes that this is true. Our freedom to doubt was born of a struggle against authority in the early days of science. It was a very deep and strong struggle. Permit us to question—to doubt, that's all—not to be sure. And I think it is important that we do not forget the importance of this struggle and thus perhaps lose what we have gained. Here lies a responsibility to society.

We are all sad when we think of the wondrous potentialities human beings seem to have, as contrasted with their small accomplishments. Again and again people have thought that we could do much better. They of the past saw in the nightmare of their times a dream for the future. We, of their future, see that their dreams, in certain ways surpassed, have in many ways remained dreams. The hopes for the future today are, in good share, those of yesterday.

EDUCATION, FOR GOOD AND EVIL

Once some thought that the possibilities people had were not developed because most of those people were ignorant. With education universal, could all men be Voltaires? Bad can be taught at least as efficiently as good. Education is a strong force, but for either good or evil.

Communications between nations must promote understanding: so went another dream. But the machines of communication can be channeled or choked. What is communicated can be truth or lie. Communication is a strong force also, but for either good or bad.

The applied sciences should free men of material problems at least. Medicine controls diseases. And the record here seems all to the good. Yet there are men patiently working to create great plagues and poisons. They are to be used in warfare tomorrow.

Nearly everybody dislikes war. Our dream today is peace. In peace, man can develop best the enormous possibilities he seems to have. But maybe future men will find that peace, too, can be good and bad. Perhaps peaceful men will drink out of boredom. Then perhaps drink will become the great problem which seems to keep man from getting all he thinks he should out of his abilities.

Clearly, peace is a great force, as is sobriety, as are material power, communication, education, honesty and the ideals of many dreamers.

We have more of these forces to control than did the ancients. And maybe we are doing a little better than most of them could do. But what we ought to be able to do seems gigantic compared with our confused accomplishments.

Why is this? Why can't we conquer ourselves?

Because we find that even great forces and abilities do not seem to carry with them clear instructions on how to use them. As an example, the great accumulation of understanding as to how the physical world behaves only convinces one that this behavior seems to have a kind of meaninglessness. The sciences do not directly teach good and bad.

Through all ages men have tried to fathom the meaning of life. They have realized that if some direction or meaning could be given to our actions, great human forces would be unleashed. So, very many answers must have been given to the question of the meaning of it all. But they have been of all different sorts, and the proponents of one answer have looked with horror at the actions of the believers in another. Horror, because from a disagreeing point of view all the great potentialities of the race were being channeled into a false and confining blind alley. In fact, it is from the history of the enormous monstrosities created by false belief that philosophers have realized the apparently infinite and wondrous capacities of human beings. The dream is to find the open channel.

What, then, is the meaning of it all? What can we say to dispel the mystery of existence?

If we take everything into account, not only what the ancients knew, but all of what we know today that they didn't know, then I think that we must frankly admit that *we do not know*.

But, in admitting this, we have probably found the open channel.

This is not a new idea; this is the idea of the age of reason. This is the philosophy that guided the men who made the

democracy that we live under. The idea that no one really knew how to run a government led to the idea that we should arrange a system by which new ideas could be developed, tried out, tossed out, more new ideas brought in; a trial and error system. This method was a result of the fact that science was already showing itself to be a successful venture at the end of the 18th century. Even then it was clear to socially-minded people that the openness of the possibilities was an opportunity, and that doubt and discussion were essential to progress into the unknown. If we want to solve a problem that we have never solved before, we must leave the door to the unknown ajar.

OUR RESPONSIBILITY AS SCIENTISTS

We are at the very beginning of time for the human race. It is not unreasonable that we grapple with problems. There are tens of thousands of years in the future. Our responsibility is to do what we can, learn what we can, improve the solutions and pass them on. It is our responsibility to leave the men of the future a free hand. In the impetuous youth of humanity, we can make grave errors that can stunt our growth for a long time. This we will do if we say we have the answers now, so young and ignorant; if we suppress all discussion, all criticism, saying, "This is it, boys, man is saved!" and thus doom man for a long time to the chains of authority, confined to the limits of our present imagination. It has been done so many times before.

It is our responsibility as scientists, knowing the great progress and great value of a satisfactory philosophy of ignorance, the great progress that is the fruit of freedom of thought, to proclaim the value of this freedom, to teach how doubt is not to be feared but welcomed and discussed, and to demand this freedom as our duty to all coming generations.

This chapter from a science fiction novel by a present-day astronomer offers some non-fiction insight into the way the scientist works. Another chapter from this same novel is in the Unit 2 Reader.

2 Close Reasoning

Fred Hoyle

A chapter from his book *The Black Cloud*, 1957.

It is curious in how great a degree human progress depends on the individual. Humans, numbered in thousands of millions, seem organised into an ant-like society. Yet this is not so. New ideas, the impetus of all development, come from individual people, not from corporations or states. New ideas, fragile as spring flowers, easily bruised by the tread of the multitude, may yet be cherished by the solitary wanderer.

Among the vast host that experienced the coming of the Cloud, none except Kingsley arrived at a coherent understanding of its real nature, none except Kingsley gave the reason for the visit of the Cloud to the solar system. His first bald statement was greeted with outright disbelief even by his fellow scientists—Alexandrov excepted.

Weichart was frank in his opinion.

"The whole idea is quite ridiculous," he said

Marlowe shook his head.

"This comes of reading science fiction."

"No bloody fiction about Cloud coming straight for dam' Sun. No bloody fiction about Cloud stopping. No bloody fiction about ionisation," growled Alexandrov.

McNeil, the physician, was intrigued. The new development was more in his line than transmitters and aerials.

"I'd like to know, Chris, what you mean in this context by the word 'alive.' "

"Well, John, you know better than I do that the distinction between animate and inanimate is more a matter of verbal convenience than anything else. By and large, inanimate matter has a simple structure and comparatively simple properties. Animate or living matter on the other hand has a highly complicated structure and is capable of very involved behaviour. When I said the Cloud may be alive I meant that the material inside it may be organised in an intricate fashion, so that its behaviour and consequently the behaviour of the whole Cloud is far more complex than we previously supposed."

"Isn't there an element of tautology there?"—from Weichart.

"I said that words such as 'animate' and 'inanimate' are only verbal conveniences. If they're pushed too far they do appear tautological. In more scientific terms I expect the chemistry of the interior of the Cloud to be extremely complicated—complicated molecules, complicated structures built out of molecules, complicated nervous activity. In short I think the Cloud has a brain."

"A dam' straightforward conclusion," nodded Alexandrov.

When the laugh had subsided, Marlowe turned to Kingsley.

"Well, Chris, we know what you mean; at any rate we know near enough. Now let's have your argument. Take your time. Let's have it point by point, and it'd better be good."

"Very well then, here goes. Point number one, the temperature inside the Cloud is suited to the formation of highly complicated molecules."

"Right! First point to you. In fact, the temperature is perhaps a little more favourable than it is here on the Earth."

"Second point, conditions are favourable to the formation of extensive structures built out of complicated molecules."

"Why should that be so?" asked Yvette Hedelfort.

"Adhesion on the surface of solid particles. The density inside the Cloud is so high that quite large lumps of solid material—probably mostly ordinary ice—are almost certainly to be found inside it. I suggest that the complicated molecules get together when they happen to stick to the surfaces of these lumps."

"A very good point, Chris," agreed Marlowe.

"Sorry, I don't pass this round." McNeil was shaking his head. "You talk of complicated molecules being built up by sticking together on the surface of solid bodies. Well, it won't do. The molecules out of which living material is made contain large stores of internal energy. Indeed, the processes of life depend on this internal energy. The trouble with your sticking together is that you don't get energy into the molecules that way."

Kingsley seemed unperturbed.

"And from what source do the molecules of living creatures here on the Earth get their internal supplies of energy?" he asked McNeil.

"Plants get it from sunlight, and animals get it from plants, or from other animals of course. So in the last analysis the energy always comes from the Sun."

"And where is the Cloud getting energy from now?"

The tables were turned. And as neither McNeil nor anyone else seemed disposed to argue, Kingsley went on:

"Let's accept John's argument. Let's suppose that my beast in the Cloud is built out of the same sort of molecules that we are. Then the light from some star is required in order that the molecules be formed. Well, of course starlight is available far out in the space between the stars, but it's very feeble. So to get a really strong supply of light the beast would need to approach close to some star. And that's just what the beast has done!"

Marlowe became excited.

"My God, that ties three things together, straight away. The need for sunlight, number one. The Cloud making a bee-line for the Sun, number two. The Cloud stopping when it reached the Sun, number three. Very good, Chris."

"It is a very good beginning, yes, but it leaves some things obscure," Yvette Hedelfort remarked. "I do not see," she went on, "how it was that the Cloud came to be out in space. If it has need of sunlight or starlight, surely it would stay always around one star. Do you suppose that this beast of yours has just been born somewhere out in space and has now come to attach itself to our Sun?"

"And while you're about it, Chris, will you explain how your friend the beast controls its supplies of energy? How did it manage to fire off those blobs of gas with such fantastic speed when it was slowing down?" asked Leicester.

"One question at a time! I'll take Harry's first, because it's probably easier. We tried to explain the expulsion of those blobs of gas in terms of magnetic fields, and the explanation simply didn't work. The trouble was that the required fields would be so intense that they'd simply burst the whole Cloud apart. Stated somewhat differently, we couldn't find any way in which large quantities of energy could be localised through a magnetic agency in comparatively small regions. But let's now look at the problem from this new point of view. Let's begin by asking what method we ourselves would use to produce intense local concentrations of energy."

"Explosions!" gasped Barnett.

"That's right, explosions, either by nuclear fission, or more probably by nuclear fusion. There's no shortage of hydrogen in this Cloud."

"Are you being serious, Chris?"

"Of course I'm being serious. If I'm right in supposing that some beast inhabits the Cloud, then why shouldn't he be at least as intelligent as we are?"

"There's the slight difficulty of radioactive products. Wouldn't these be extremely deleterious to living material?" asked McNeil.

"If they could get at the living material, certainly they would. But although it isn't possible to produce explosions with magnetic fields, it is possible to prevent two samples of

material mixing with each other. I imagine that the beast orders the material of the Cloud magnetically, that by means of magnetic fields he can move samples of material wherever he wants inside the Cloud. I imagine that he takes very good care to keep the radioactive gas well separated from the living material—remember I'm using the term 'living' for verbal convenience. I'm not going to be drawn into a philosophical argument about it."

"You know, Kingsley," said Weichart, "this is going far better than I thought it would. What I suppose you would say is that whereas basically we assemble materials with our hands, or with the aid of machines that we have made with our hands, the beast assembles materials with the aid of magnetic energy."

"That's the general idea. And I must add that the beast seems to me to have far the better of it. For one thing he's got vastly more energy to play with than we have."

"My God, I should think so, billions of times more, at the very least," said Marlowe. "It's beginning to look, Chris, as if you're winning this argument. But we objectors over here in this corner are pinning our faith to Yvette's question. It seems to me a very good one. What can you offer in answer to it?"

"It is a very good question, Geoff, and I don't know that I can give a really convincing answer. The sort of idea I've got is that perhaps the beast can't stay for very long in the close proximity of a star. Perhaps he comes in periodically to some star or other, builds his molecules, which form his food supply as it were, and then pushes off again. Perhaps he does this time and time again."

"But why shouldn't the beast be able to stay permanently near a star?"

"Well, an ordinary common or garden cloud, a beastless cloud, if it were permanently near a star, would gradually condense into a compact body, or into a number of compact bodies. Indeed, as we all know, our Earth probably condensed at one time from just such a cloud. Obviously our friend the beast would find it extremely embarrassing to have his protective Cloud condense into a planet. So equally obviously he'll decide to push off before there's any danger of that happening. And when he pushes off he'll take his Cloud with him."

"Have you any idea of how long that will be?" asked Parkinson.

"None at all. I suggest that the beast will push off when he's finished recharging his food supply. That might be a matter of weeks, months, years, millennia for all I know."

"Don't I detect a slight smell of rat in all this?" Barnett remarked.

"Possibly. I don't know how keen your sense of smell is, Bill. What's your trouble?"

"I've got lots of troubles. I should have thought that your remarks about condensing into a planet apply only to an inanimate cloud. If we grant that the Cloud is able to control the distribution of material within itself, then it could easily prevent condensation from taking place. After all, condensation must be a sort of stability process and I would have thought that quite a moderate degree of control on the part of your beast could prevent any condensation at all."

"There are two replies to that. One is that I believe the beast will lose his control if he stays too long near the Sun. If he stays too long, the magnetic field of the Sun will penetrate into the Cloud. Then the rotation of the Cloud round the Sun will twist up the magnetic field to blazes. All control would then be lost."

"My God, that's an excellent point."

"It is, isn't it? And here's another one. However different our beast is to life here on Earth, one point he must have in common with us. We must both obey the simple biological rules of selection and development. By that I mean that we can't suppose that the Cloud started off by containing a fully-fledged beast. It must have started with small beginnings, just as life here on Earth started with small beginnings. So to start with there would be no intricate control over the distribution of material in the Cloud. Hence if the Cloud had originally been situated close to a star, it could not have prevented condensation into a planet or into a number of planets."

"Then how do you visualise the early beginnings?"

"As something that happened far out in interstellar space. To begin with, life in the Cloud must have depended on the general radiation field of the stars. Even that would give it more radiation for molecule-building purposes than life on the Earth gets. Then I imagine that as intelligence developed it would be discovered that food supplies—i.e. molecule-building—could be enormously increased by moving in close to a star for a comparatively brief period. As I see it, the beast must be essentially a denizen of interstellar space. Now, Bill, have you any more troubles?"

"Well, yes, I've got another problem. Why can't the Cloud manufacture its own radiation? Why bother to come in close to a star? If it understands nuclear fusion to the point of producing gigantic explosions, why not use nuclear fusion for producing its supply of radiation?"

"To produce radiation in a controlled fashion requires a slow reactor, and of course that's just what a star is. The Sun is just a gigantic slow nuclear fusion reactor. To produce radiation on any real scale comparable with the Sun, the Cloud would have to make itself into a star. Then the beast would get roasted. It'd be much too hot inside."

"Even then I doubt whether a cloud of this mass could produce very much radiation," remarked Marlowe. "Its mass is much too small. According to the mass-luminosity

relation it'd be down as compared with the Sun by a fantastic amount. No, you're barking up a wrong tree there, Bill."

"I've a question that I'd like to ask," said Parkinson. "Why do you always refer to your beast in the singular? Why shouldn't there be lots of little beasts in the Cloud?"

"I have a reason for that, but it'll take quite a while to explain."

"Well, it looks as if we're not going to get much sleep tonight, so you'd better carry on."

"Then let's start by supposing that the Cloud contains lots of little beasts instead of one big beast. I think you'll grant me that communication must have developed between the different individuals."

"Certainly."

"Then what form will the communication take?"

"You're supposed to be telling us, Chris."

"My question was purely rhetorical. I suggest that communication would be impossible by our methods. We communicate acoustically."

"You mean by talking. That's certainly your method all right, Chris," said Ann Halsey.

But the point was lost on Kingsley. He went on.

"Any attempt to use sound would be drowned by the enormous amount of background noise that must exist inside the Cloud. It would be far worse than trying to talk in a roaring gale. I think we can be pretty sure that communication would have to take place electrically."

"That seems fair enough."

"Good. Well, the next point is that by our standards the distances between the individuals would be very large, since the Cloud by our standards is enormously large. It would obviously be intolerable to rely on essentially D.C. methods over such distances."

"D.C. methods? Chris, will you please try to avoid jargon."

"Direct current."

"That explains it, I suppose!"

"Oh, the sort of thing we get on the telephone. Roughly speaking the difference between D.C. communication and A.C. communication is the difference between the telephone and radio."

Marlowe grinned at Ann Halsey.

"What Chris is trying to say in his inimitable manner is that communication must occur by radiative propagation."

"If you think that makes it clearer. . . ."

"Of course it's clear. Stop being obstructive, Ann. Radiative propagation occurs when we emit a light signal or a radio signal. It travels across space through a vacuum at a speed of 186,000 miles per second. Even at this speed it would still take about ten minutes for a signal to travel across the Cloud.

"My next point is that the volume of information that can be transmitted radiatively is enormously greater than the amount that we can communicate by ordinary sound. We've seen that with our pulsed radio transmitters. So if this Cloud contains separate individuals, the individuals must be able to communicate on a vastly more detailed scale than we can. What we can get across in an hour of talk they might get across in a hundredth of a second."

"Ah, I begin to see light," broke in McNeil. "If communication occurs on such a scale then it becomes somewhat doubtful whether we should talk any more of separate individuals!"

"You're home, John!"

"But *I'm* not home," said Parkinson.

"In vulgar parlance," said McNeil amiably, "what Chris is saying is that individuals in the Cloud, if there are any, must be highly telepathic, so telepathic that it becomes rather meaningless to regard them as being really separate from each other."

"Then why didn't he say so in the first place?"—from Ann Halsey.

"Because like most vulgar parlance, the word 'telepathy' doesn't really mean very much."

"Well, it certainly means a great deal more to me."

"And what does it mean to you, Ann?"

"It means conveying one's thoughts without talking, or of course without writing or winking or anything like that."

"In other words it means—if it means anything at all—communication by a non-acoustic medium."

"And that means using radiative propagation," chipped in Leicester.

"And radiative propagation means the use of alternating currents, not the direct currents and voltages we use in our brains."

"But I thought we were capable of some degree of telepathy," suggested Parkinson.

"Rubbish. Our brains simply don't work the right way for telepathy. Everything is based on D.C. voltages, and radiative transmission is impossible that way."

"I know this is rather a red herring, but I thought these extrasensory people had established some rather remarkable correlations," Parkinson persisted.

"Bloody bad science," growled Alexandrov. "Correlations obtained after experiments done is bloody bad. Only prediction in science."

"I don't follow."

"What Alexis means is that only predictions really count in science," explained Weichart. "That's the way Kingsley downed me an hour or two ago. It's no good doing a lot of experiments first and then discovering a lot of correlations afterwards, not unless the correlations can be used for

making new predictions. Otherwise it's like betting on a race after it's been run."

"Kingsley's ideas have many very interesting neurological implications," McNeil remarked. "Communication for us is a matter of extreme difficulty. We ourselves have to make a translation of the electrical activity—essentially D.C. activity—in our brains. To do this quite a bit of the brain is given over to the control of the lip muscles and of the vocal cords. Even so our translation is very incomplete. We don't do too badly perhaps in conveying simple ideas, but the conveying of emotions is very difficult. Kingsley's little beasts could, I suppose, convey emotions too, and that's another reason why it's rather meaningless to talk of separate individuals. It's rather terrifying to realise that everything we've been talking about tonight and conveying so inadequately from one to another could be communicated with vastly greater precision and understanding among Kingsley's little beasts in about a hundredth of a second."

"I'd like to follow the idea of separate individuals a little further," said Barnett, turning to Kingsley. "Would you think of each individual in the Cloud as building a radiative transmitter of some sort?"

"Not as *building* a transmitter. Let me describe how I see biological evolution taking place within the Cloud. At an early stage I think there would be a whole lot of more or less separate disconnected individuals. Then communication would develop, not by a deliberate inorganic building of a means of radiative transmission, but through a slow biological development. The individuals would develop a means of radiative transmission as a biological organ, rather as we have developed a mouth, tongue, lips, and vocal cords. Communication would improve to a degree that we can scarcely contemplate. A thought would no sooner be thought than it would be communicated. An emotion would no sooner be experienced than it would be shared. With this would come a submergence of the individual and an evolution into a coherent whole. The beast, as I visualise it, need not be located in a particular place in the Cloud. Its different parts may be spread through the Cloud, but I regard it as a neurological unity, interlocked by a communication system in which signals are transmitted back and forth at a speed of 186,000 miles a second."

"We ought to get down to considering those signals more closely. I suppose they'd have to have a longish wave-length. Ordinary light presumably would be useless since the Cloud is opaque to it," said Leicester.

"My guess is that the signals are radio waves," went on Kingsley. "There's a good reason why it should be so. To be really efficient one must have complete phase control in a communication system. This can be done with radio waves, but not so far as we know with shorter wave-lengths."

McNeil was excited.

"Our radio transmissions!" he exclaimed. "They'd have interfered with the beast's neurological control."

"They would if they'd been allowed to."

"What d'you mean, Chris?"

"Well, the beast hasn't only to contend with our transmissions, but with the whole welter of cosmic radio waves. From all quarters of the Universe there'd be radio waves interfering with its neurological activity unless it had developed some form of protection."

"What sort of protection have you in mind?"

"Electrical discharges in the outer part of the Cloud causing sufficient ionisation to prevent the entry of external radio waves. Such a protection would be as essential as the skull is to the human brain."

Aniseed smoke was rapidly filling the room. Marlowe suddenly found his pipe too hot to hold and put it down gingerly.

"My God, you think this explains the rise of ionisation in the atmosphere, when we switch on our transmitters?"

"That's the general idea. We were talking earlier on about a feedback mechanism. That I imagine is just what the beast has got. If any external waves get in too deeply, then up go the voltages and away go the discharges until the waves can get in no farther."

"But the ionisation takes place in our own atmosphere."

"For this purpose I think we can regard our atmosphere as a part of the Cloud. We know from the shimmering of the night sky that gas extends all the way from the Earth to the denser parts of the Cloud, the disk-like parts. In short we're inside the Cloud, electronically speaking. That, I think, explains our communication troubles. At an earlier stage, when we were outside the Cloud, the beast didn't protect itself by ionising our atmosphere, but through its outer electronic shield. But once we got inside the shield the discharges began to occur in our own atmosphere. The beast has been boxing-in our transmissions."

"Very fine reasoning, Chris," said Marlowe.

"Hellish fine," nodded Alexandrov.

"How about the one centimetre transmissions? They went through all right," Weichart objected.

"Although the chain of reasoning is getting rather long there's a suggestion that one can make on that. I think it's worth making because it suggests the next action we might take. It seems to me most unlikely that this Cloud is unique. Nature doesn't work in unique examples. So let's suppose there are lots of these beasts inhabiting the Galaxy. Then I would expect communication to occur between one cloud and another. This would imply that some wavelengths would be required for external communication pur-

poses, wave-lengths that could penetrate into the Cloud and would do no neurological harm."

"And you think one centimetre may be such a wavelength?"

"That's the general idea."

"But then why was there no reply to our one centimetre transmission?" asked Parkinson.

"Perhaps because we sent no message. There'd be no point in replying to a perfectly blank transmission."

"Then we ought to start sending pulsed messages on the one centimetre," exclaimed Leicester. "But how can we expect the Cloud to decipher them?"

"That's not an urgent problem to begin with. It will be obvious that our transmissions contain information—that will be clear from the frequent repetition of various patterns. As soon as the Cloud realises that our transmissions have intelligent control behind them I think we can expect some sort of reply. How long will it take to get started, Harry? You're not in a position to modulate the one centimetre yet, are you."

"No, but we can be in a couple of days, if we work night shifts. I had a sort of presentiment that I wasn't going to see my bed tonight. Come on, chaps, let's get started."

Leicester stood up, stretched himself, and ambled out. The meeting broke up. Kingsley took Parkinson on one side.

"Look, Parkinson," he said, "there's no need to go gabbling about this until we know more about it."

"Of course not. The Prime Minister suspects I'm off my head as it is."

"There is one thing that you might pass on, though. If London, Washington, and the rest of the political circus could get ten centimetre transmitters working, it's just possible that they might avoid the fade-out trouble."

When Kingsley and Ann Halsey were alone later that night, Ann remarked:

"How on earth did you come on such an idea, Chris?"

"Well, it's pretty obvious really. The trouble is that we're all inhibited against such thinking. The idea that the Earth is the only possible abode of life runs pretty deep in spite of all the science fiction and kid's comics. If we had been able to look at the business with an impartial eye we should have spotted it long ago. Right from the first, things have gone wrong and they've gone wrong according to a systematic sort of pattern. Once I overcame the psychological block, I saw all the difficulties could be removed by one simple and entirely plausible step. One by one the bits of the puzzle fitted into place. I think Alexandrov probably had the same idea, only his English is a bit on the terse side."

"On the bloody terse side, you mean. But seriously, do you think this communication business will work?"

"I very much hope so. It's quite crucial that it should."

"Why do you say that?"

"Think of the disasters the Earth has suffered so far, without the Cloud taking any purposive steps against us. A bit of reflection from its surface nearly roasted us. A short obscuration of the Sun nearly froze us. If the merest tiny fraction of the energy controlled by the Cloud should be directed against us we should be wiped out, every plant and animal."

"But why should that happen?"

"How can one tell? Do you think of the tiny beetle or the ant that you crush under your foot on an afternoon's walk? One of those gas bullets that hit the Moon three months ago would finish us. Sooner or later the Cloud will probably let fly with some more of 'em. Or we might be electrocuted in some monstrous discharge."

"Could the Cloud really do that?"

"Easily. The energy that it controls is simply monstrous. If we can get some sort of a message across, then perhaps the Cloud will take the trouble to avoid crushing us under its foot."

"But why should it bother?"

"Well, if a beetle were to say to you, 'Please, Miss Halsey, will you avoid treading here, otherwise I shall be crushed,' wouldn't you be willing to move your foot a trifle?"

Scientists often stress that there is no single scientific method. Bridgman emphasizes this freedom to choose between many procedures, a freedom essential to science.

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3 On Scientific Method

Percy W. Bridgman

An excerpt from his book *Reflections of a Physicist*, 1955.

IT SEEMS TO ME that there is a good deal of ballyhoo about scientific method. I venture to think that the people who talk most about it are the people who do least about it. Scientific method is what working scientists do, not what other people or even they themselves may say about it. No working scientist, when he plans an experiment in the laboratory, asks himself whether he is being properly scientific, nor is he interested in whatever method he may be using *as method*. When the scientist ventures to criticize the work of his fellow scientist, as is not uncommon, he does not base his criticism on such glittering generalities as failure to follow the "scientific method," but his criticism is specific, based on some feature characteristic of the particular situation. The working scientist is always too much concerned with getting down to brass tacks to be willing to spend his time on generalities.

Scientific method is something talked about by people standing on the outside and wondering how the scientist manages to do it. These people have been able to uncover various generalities applicable to at least most of what the scientist does, but it seems to me that these generalities are not very profound, and could have been anticipated by anyone who knew enough about scientists to know what is their primary objective. I think that the objectives of all scientists have this in common—that they are

all trying to get the correct answer to the particular problem in hand. This may be expressed in more pretentious language as the pursuit of truth. Now if the answer to the problem is correct there must be some way of knowing and proving that it is correct —the very meaning of truth implies the possibility of checking or verification. Hence the necessity for checking his results always inheres in what the scientist does. Furthermore, this checking must be exhaustive, for the truth of a general proposition may be disproved by a single exceptional case. A long experience has shown the scientist that various things are inimical to getting the correct answer. He has found that it is not sufficient to trust the word of his neighbor, but that if he wants to be sure, he must be able to check a result for himself. Hence the scientist is the enemy of all authoritarianism. Furthermore, he finds that he often makes mistakes himself and he must learn how to guard against them. He cannot permit himself any preconception as to what sort of results he will get, nor must he allow himself to be influenced by wishful thinking or any personal bias. All these things together give that "objectivity" to science which is often thought to be the essence of the scientific method.

But to the working scientist himself all this appears obvious and trite. What appears to him as the essence of the situation is that he is not consciously following any prescribed course of action, but feels complete freedom to utilize any method or device whatever which in the particular situation before him seems likely to yield the correct answer. In his attack on his specific problem he suffers no inhibitions of precedent or authority, but is completely free to adopt any course that his ingenuity is capable of suggesting to him. No one standing on the outside can predict what the individual scientist will do or what method he will follow. In short, science is what scientists do, and there are as many scientific methods as there are individual scientists.

This is Polya's one-page summary of his book in which he discusses strategies and techniques for solving problems. Polya's examples are from mathematics, but his ideas are useful in solving physics problems also.

4 How to Solve It

George Polya

An excerpt from his book *How To Solve It*, 1945.

UNDERSTANDING THE PROBLEM

First.

You have to *understand* the problem.

What is the unknown? What are the data? What is the condition? Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it insufficient? Or redundant? Or contradictory?

Draw a figure. Introduce suitable notation.

Separate the various parts of the condition. Can you write them down?

DEVISING A PLAN

Second.

Find the connection between the data and the unknown. You may be obliged to consider auxiliary problems if an immediate connection cannot be found. You should obtain eventually a *plan* of the solution.

Have you seen it before? Or have you seen the same problem in a slightly different form?

Do you know a related problem? Do you know a theorem that could be useful?

Look at the unknown! And try to think of a familiar problem having the same or a similar unknown.

Here is a problem related to yours and solved before. Could you use it? Could you use its result? Could you use its method? Should you introduce some auxiliary element in order to make its use possible?

Could you restate the problem? Could you restate it still differently? Go back to definitions.

If you cannot solve the proposed problem try to solve first some related problem. Could you imagine a more accessible related problem? A more general problem? A more special problem? An analogous problem? Could you solve a part of the problem? Keep only a part of the condition, drop the other part; how far is the unknown then determined, how can it vary? Could you derive something useful from the data? Could you think of other data appropriate to determine the unknown? Could you change the unknown or the data, or both if necessary, so that the new unknown and the new data are nearer to each other?

Did you use all the data? Did you use the whole condition? Have you taken into account all essential notions involved in the problem?

CARRYING OUT THE PLAN

Third.

Carry out your plan.

Carrying out your plan of the solution, *check each step*. Can you see clearly that the step is correct? Can you prove that it is correct?

LOOKING BACK

Fourth.

Examine the solution obtained.

Can you *check the result?* Can you check the argument?

Can you derive the result differently? Can you see it at a glance?

Can you use the result, or the method, for some other problem?

The advice is directed primarily to the student planning a career in the sciences, but it should be of interest to a wider group.

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5 Four Pieces of Advice to Young People

Warren Weaver

Part of a talk given in Seattle during the Arches of Science Award Seminars, 1966.

One of the great prerogatives of age is the right to give advice to the young. Of course, the other side of the coin is that one of the prerogatives of youth is to disregard this advice. But...I am going to give you four pieces of advice, and you may do with all four of them precisely what you see fit.

The first one is this: I urge each one of you not to decide prematurely what field of science, what specialty of science you are going to make your own. Science moves very rapidly. Five years from now or ten years from now there will be opportunities in science which are almost not discernible at the present time. And, I think there are also, of course, fads in science. Science goes all out at any one moment for work in one certain direction and the other fields are thought of as being rather old-fashioned. But, don't let that fool you. Sometimes some of these very old problems turn out to be extremely significant.

May I just remind you that there is no physical entity that the mind of man has thought about longer than the phenomenon of light. One would ordinarily say that it would be simply impossible at the present day for someone to sit down and get a brand new idea about light, because think of the thousands of scientists that have worked on that subject. And yet, you see this is what two scientists did only just a few years ago when the laser was invented. They got a brand new idea about light and it has turned out to be a phenomenally important idea.

So, I urge you not to make up your minds too narrowly, too soon. Of course, that means that what you ought to do is to be certain that you get a very solid basic foundation in science so that you can then adjust yourselves to the opportunities of the future when they arise. What is that basic foundation?

Well, of course, you don't expect me to say much more than mathematics, do you? Because I was originally trained as a mathematician and mathematics is certainly at the bottom of all this. But I also mean the fundamentals of physics and the fundamentals of chemistry. These two, incidentally, are almost indistinguishable nowadays from the fundamentals of biology.

The secondpieceof advice that I will just mention to you because maybe some of you are thinking too exclusively in terms of a career in research. In my judgment there is no life that is possible to be lived on this planet that is more pleasant and more rewarding than the combined activity of teaching and research.

I hope very much that many of you look forward to becoming teachers. It is a wonderful life. I don't know of any better one myself, any more pleasant one, or any more rewarding one. And the almost incredible fact is that they even pay you for it. And, nowadays, they don't pay you too badly. Of course, when I started, they did. But, nowadays, the pay is pretty good.

My third piece of advice—may I urge every single one of you to prepare yourself not only to be a scientist, but to be a scientist-citizen. You have to accept the responsibilities of citizenship in a free democracy. And those are great responsibilities and because of the role which science plays in our modern world, we need more and more people who understand science but who are also sensitive to and aware of the responsibilities of citizenship.

And the final piece of advice is—and maybe this will surprise you: Do not overestimate science, do not think that science is all that there is, do not concentrate so completely on science that you end up by living a warped sort of life. Science is not all that there is, and science is not capable of solving all of life's problems. There are also many more very important problems that science cannot solve.

And so I hope very much there's nobody in this room who is going to spend the next seven days without reading some poetry. I hope that there's nobody in this room that's going to spend the next seven days without listening to some music, some good music, some modern music, some music. I hope very much that there's nobody here who is not interested in the creative arts, interested in drama, interested in the dance. I hope that you interest yourselves seriously in religion, because if you do not open your minds and open your activities to this range of things, you are going to lead too narrow a life.

The size of an animal is related to such physical factors as gravity and temperature. For most animals there appears to be an optimum size.

6 On Being the Right Size

J. B. S. Haldane

An excerpt from his book *Possible Worlds*, 1928.

> *From what has already been demonstrated, you can plainly see the impossibility of increasing the size of structures to vast dimensions either in art or in nature; likewise the impossibility of building ships, palaces, or temples of enormous size in such a way that their oars, yards, beams, iron bolts, and, in short, all their other parts will hold together; nor can nature produce trees of extraordinary size because the branches would break down under their own weight, so also it would be impossible to build up the bony structures of men, horses, or other animals so as to hold together and perform their normal functions if these animals were to be increased enormously in height; for this increase in height can be accomplished only by employing a material which is harder and stronger than usual, or by enlarging the size of the bones, thus changing their shape until the form and appearance of the animals suggest a monstrosity. This is perhaps what our wise Poet had in mind, when he says, in describing a huge giant:*
> *"Impossible it is to reckon his height*
> *So beyond measure is his size."* —Galileo Galilei

THE most obvious differences between different animals are differences of size, but for some reason the zoologists have paid singularly little attention to them. In a large textbook of zoology before me I find no indication that the eagle is larger than the sparrow, or the hippopotamus bigger than the hare, though some grudging admissions are made in the case of the mouse and the whale. But yet it is easy to show that a hare could not be as large as a hippopotamus, or a whale as small as a herring. For every type of animal there is a most convenient size, and a large change in size inevitably carries with it a change of form.

Let us take the most obvious of possible cases, and consider a giant man sixty feet high—about the height of Giant Pope and Giant Pagan in the illustrated *Pilgrim's Progress* of my childhood. These monsters were not only ten times as high as Christian, but ten times as wide and ten times as thick, so that their total weight was a thousand times his, or about eighty to ninety tons. Unfortunately the cross sections of their bones were only a hundred times those of Christian, so that every square inch of giant bone had to support ten times the weight borne by a square inch of human bone. As the human thigh-bone breaks under about ten times the human weight, Pope and Pagan would have broken their thighs every time they took a step. This was doubtless why they were sitting down in the picture I remember. But it lessens one's respect for Christian and Jack the Giant Killer.

To turn to zoology, suppose that a gazelle, a graceful little creature with long thin legs, is to become large, it will break its bones unless it does one of two things. It may make its legs short and thick, like the rhinoceros, so that every pound of weight has still about the same area of bone to support it. Or it can compress its body and stretch out its legs obliquely to gain stability, like the giraffe. I mention these two beasts because they happen to belong to the same order as the gazelle, and both are quite successful mechanically, being remarkably fast runners.

Gravity, a mere nuisance to Christian, was a terror to Pope, Pagan, and Despair. To the mouse and any smaller animal it presents practically no dangers. You can drop a mouse down a thousand-yard mine shaft; and, on arriving at the bottom, it gets a slight shock and walks away. A rat would probably be killed, though it can fall safely from the eleventh story of a building; a man is killed, a horse splashes. For the resistance presented to movement by the air is proportional to the surface of the moving object. Divide an animal's length, breadth, and height each by ten; its weight is reduced to a thousandth, but its surface only to a hundredth. So the resistance to falling in the case of the small animal is relatively ten times greater than the driving force.

An insect, therefore, is not afraid of gravity; it can fall without danger, and can cling to the ceiling with remarkably little trouble. It can go in for elegant and fantastic forms of support like that of the daddy-long-legs. But there is a force which is as formidable to an insect as gravitation to a mammal. This is surface tension. A man coming out of a bath carries with him a film of water of about one-fiftieth of an inch in thickness. This weighs roughly a pound. A wet mouse has to carry about its own weight of water. A wet fly has to lift many times its own weight and, as every one knows, a fly once wetted by water or any other liquid is in a very serious position indeed. An insect going for a drink is in as great danger as a man leaning out over a precipice in search of food. If it once falls into the grip of the surface tension of the water—that is to say, gets wet—it is likely to remain so until it drowns. A few insects, such as water-beetles, contrive to be unwettable, the majority keep well away from their drink by means of a long proboscis.

Of course tall land animals have other difficulties. They have to pump their blood to greater heights than a man and, therefore, require a larger blood pressure and tougher blood-vessels. A great many men die from burst arteries, especially in the brain, and this danger is presumably still greater for an elephant or a giraffe. But animals of all kinds find difficulties in size for the following reason. A typical small animal, say a microscopic worm or rotifer, has a smooth skin through which all the oxygen it requires can soak in, a straight gut with sufficient surface to absorb its food, and a simple kidney. Increase its dimensions tenfold in every direction, and its weight is increased a thousand times, so that if it is to use its muscles as efficiently as its miniature counterpart, it will need a thousand times as much food and oxygen per day and will excrete a thousand times as much of waste products.

Now if its shape is unaltered its surface will be increased only a hundredfold, and ten times as much oxygen must enter per minute through

each square millimetre of skin, ten times as much food through each square millimetre of intestine. When a limit is reached to their absorptive powers their surface has to be increased by some special device. For example, a part of the skin may be drawn out into tufts to make gills or pushed in to make lungs, thus increasing the oxygen-absorbing surface in proportion to the animal's bulk. A man, for example, has a hundred square yards of lung. Similarly, the gut, instead of being smooth and straight, becomes coiled and develops a velvety surface, and other organs increase in complication. The higher animals are not larger than the lower because they are more complicated. They are more complicated because they are larger. Just the same is true of plants. The simplest plants, such as the green algae growing in stagnant water or on the bark of trees, are mere round cells. The higher plants increase their surface by putting out leaves and roots. Comparative anatomy is largely the story of the struggle to increase surface in proportion to volume.

Some of the methods of increasing the surface are useful up to a point, but not capable of a very wide adaptation. For example, while vertebrates carry the oxygen from the gills or lungs all over the body in the blood, insects take air directly to every part of their body by tiny blind tubes called tracheae which open to the surface at many different points. Now, although by their breathing movements they can renew the air in the outer part of the tracheal system, the oxygen has to penetrate the finer branches by means of diffusion. Gases can diffuse easily through very small distances, not many times larger than the average length travelled by a gas molecule between collisions with other molecules. But when such vast journeys—from the point of view of a molecule—as a quarter of an inch have to be made, the process becomes slow. So the portions of an insect's body more than a quarter of an inch from the air would always be short of oxygen. In consequence hardly any insects are much more than half an inch thick. Land crabs are built on the same general plan as insects, but are much clumsier. Yet like ourselves they carry oxygen around in their blood, and are therefore able to grow far larger than any insects. If the insects had hit on a plan for driving air through their tissues instead of letting it soak in, they might well have become as large as lobsters, though other considerations would have prevented them from becoming as large as man.

Exactly the same difficulties attach to flying. It is an elementary principle of aeronautics that the minimum speed needed to keep an aeroplane of a given shape in the air varies as the square root of its length. If its linear dimensions are increased four times, it must fly twice as fast. Now the power needed for the minimum speed increases more rapidly than the weight of the machine. So the larger aeroplane, which weighs sixty-four times as much as the smaller, needs one hundred and twenty-eight times its horsepower to keep up. Applying the same principles to the birds, we find that the limit to their size is soon reached. An angel whose muscles developed no more power weight for weight than those of an eagle or a pigeon would require a breast projecting for about four feet to house the muscles engaged in working its wings, while to economize in weight, its legs would have to be reduced to mere stilts. Actually a large bird such as

an eagle or kite does not keep in the air mainly by moving its wings. It is generally to be seen soaring, that is to say balanced on a rising column of air. And even soaring becomes more and more difficult with increasing size. Were this not the case eagles might be as large as tigers and as formidable to man as hostile aeroplanes.

But it is time that we passed to some of the advantages of size. One of the most obvious is that it enables one to keep warm. All warm-blooded animals at rest lose the same amount of heat from a unit area of skin, for which purpose they need a food-supply proportional to their surface and not to their weight. Five thousand mice weigh as much as a man. Their combined surface and food or oxygen consumption are about seventeen times a man's. In fact a mouse eats about one quarter its own weight of food every day, which is mainly used in keeping it warm. For the same reason small animals cannot live in cold countries. In the arctic regions there are no reptiles or amphibians, and no small mammals. The smallest mammal in Spitzbergen is the fox. The small birds fly away in the winter, while the insects die, though their eggs can survive six months or more of frost. The most successful mammals are bears, seals, and walruses.

Similarly, the eye is a rather inefficient organ until it reaches a large size. The back of the human eye on which an image of the outside world is thrown, and which corresponds to the film of a camera, is composed of a mosaic of 'rods and cones' whose diameter is little more than a length of an average light wave. Each eye has about half a million, and for two objects to be distinguishable their images must fall on separate rods or cones. It is obvious that with fewer but larger rods and cones we should see less distinctly. If they were twice as broad two points would have to be twice as far apart before we could distinguish them at a given distance. But if their size were diminished and their number increased we should see no better. For it is impossible to form a definite image smaller than a wave-length of light. Hence a mouse's eye is not a small-scale model of a human eye. Its rods and cones are not much smaller than ours, and therefore there are far fewer of them. A mouse could not distinguish one human face from another six feet away. In order that they should be of any use at all the eyes of small animals have to be much larger in proportion to their bodies than our own. Large animals on the other hand only require relatively small eyes, and those of the whale and elephant are little larger than our own.

For rather more recondite reasons the same general principle holds true of the brain. If we compare the brain-weights of a set of very similar animals such as the cat, cheetah, leopard, and tiger, we find that as we quadruple the body-weight the brain-weight is only doubled. The larger animal with proportionately larger bones can economize on brain, eyes, and certain other organs.

Such are a very few of the considerations which show that for every type of animal there is an optimum size. Yet although Galileo demonstrated the contrary more than three hundred years ago, people still believe that if a flea were as large as a man it could jump a thousand feet into the air. As a matter of fact the height to which an animal can jump is more nearly independent of its size than proportional to it. A flea can

jump about two feet, a man about five. To jump a given height, if we neglect the resistance of the air, requires an expenditure of energy proportional to the jumper's weight. But if the jumping muscles form a constant fraction of the animal's body, the energy developed per ounce of muscle is independent of the size, provided it can be developed quickly enough in the small animal. As a matter of fact an insect's muscles, although they can contract more quickly than our own, appear to be less efficient; as otherwise a flea or grasshopper could rise six feet into the air.

And just as there is a best size for every animal, so the same is true for every human institution. In the Greek type of democracy all the citizens could listen to a series of orators and vote directly on questions of legislation. Hence their philosophers held that a small city was the largest possible democratic state. The English invention of representative government made a democratic nation possible, and the possibility was first realized in the United States, and later elsewhere. With the development of broadcasting it has once more become possible for every citizen to listen to the political views of representative orators, and the future may perhaps see the return of the national state to the Greek form of democracy. Even the referendum has been made possible only by the institution of daily newspapers.

To the biologist the problem of socialism appears largely as a problem of size. The extreme socialists desire to run every nation as a single business concern. I do not suppose that Henry Ford would find much difficulty in running Andorra or Luxembourg on a socialistic basis. He has already more men on his pay-roll than their population. It is conceivable that a syndicate of Fords, if we could find them, would make Belgium Ltd. or Denmark Inc. pay their way. But while nationalization of certain industries is an obvious possibility in the largest of states, I find it no easier to picture a completely socialized British Empire or United States than an elephant turning somersaults or a hippopotamus jumping a hedge.

Not only the scientist is interested in motion. This article comments briefly on references to motion in poetry.

7 Motion in Words

James B. Gerhart and Rudi H. Nussbaum

An excerpt from their monograph, *Motion,* 1966.

Man began describing movement with words long before there were physicists to reduce motion to laws. Our age-old fascination with moving things is attested to by the astonishing number of words we have for motion. We have all kinds of words for all kinds of movement: special words for going up, others for coming down; words for fast motion, words for slow motion. A thing going up may rise, ascend, climb, or spring. Going down again, it may fall or descend; sink, subside, or settle; dive or drop; plunge or plop; topple, totter, or merely droop. It may twirl, whirl, turn and circle; rotate, gyrate; twist or spin; roll, revolve and wheel. It may oscillate, vibrate, tremble and shake; tumble or toss, pitch or sway; flutter, jiggle, quiver, quake; or lurch, or wobble, or even palpitate. All these words tell some motion, yet every one has its own character. Some of them you use over and over in a single day. Others you may merely recognize. And still they are but a few of our words for motion. There are special words for the motions of particular things. Horses, for example, trot and gallop and canter while men run, or stride, or saunter. Babies crawl and creep. Tides ebb and flow, balls bounce, armies march. Other words tell the quality of motion, words like swift or fleet, like calm and slow.

Writers draw vivid mental pictures for the reader with words alone. Here is a poet's description of air flowing across a field on a hot day:

> There came a wind like a bugle:
> It quivered through the grass,
> and a green chill upon the heat
> so ominous did pass.
>
> Emily Dickinson

Or again, the motion of the sea caused by the gravitational attraction of the moon:

> The western tide crept up along
> the sand,
> and o'er and o'er the sand,
> and round and round the sand,
> as far as the eye could see.
>
> Charles Kingsley,
> The Sands of Dee

Or, swans starting into flight:

> I saw . . . all suddenly mount
> and scatter wheeling in great
> broken rings
> upon their clamorous wings.
>
> W. B. Yeats,
> The Wild Swans at Coole

Sometimes just a single sentence will convey the whole idea of motion:

> Lightly stepped a yellow star
> to its lofty place
>
> Emily Dickinson

Or, this description of a ship sailing:

> She walks the water like a thing
> of life
>
> Byron, The Corsair

How is it that these poets describe motion? They recall to us what we have seen; they compare different things through simile and metaphor; they rely on the reader to share their own emotions, and they invite him to recreate an image of motion in his own mind. The poet has his own precision which is not the scientist's precision. Emily Dickinson well knew it was the grass, not the wind, that quivered, and that stars don't step. Byron never saw a walking boat. But this is irrelevant. All of us can appreciate and enjoy their rich images and see that they are true to the nature of man's perception, if not to the nature of motion itself.

From time to time a physicist reading poetry will find a poem which describes something that he has learned to be of significance to his, the physicist's description. Here is an example:

> A ball will bounce, but less and less. It's not a light-hearted thing, resents its own resilience. Falling is what it loves, . . .
>
> Richard Wilbur, Juggler

Relativity is implicit in this next example:

> The earth revolves with me, yet makes no motion.
> The stars pale silently in a coral sky.
> In a whistling void I stand before my mirror unconcerned, and tie my tie.
>
> Conrad Aiken, Morning Song of Senlin

The poet's description of motion is a rich, whole vision, filled with both his perceptions and his responses. Yet complete as it is, the poetic description is far from the scientific one. Indeed, when we compare them, it is easy to forget they deal with the same things. Just how does the scientific view of motion differ? And to what purpose? Let's try to answer these questions by shifting slowly from the poet's description to the scientist's. As a first step, read this excerpt from a biography of a physicist of the last century, Lord Kelvin. The biographer is trying to convey the electric quality of Kelvin's lectures to his University classes. He describes a lecture on tops (referred to as gyrostats here):

Fig. 1.10 Multiple-flash photograph showing the precession of a top.

> The vivacity and enthusiasm of the Professor at that time was very great. The animation of his countenance as he looked at a gyrostat spinning, standing on a knife edge on a glass plate in front of him, and leaning over so that its center of gravity was on one side of the point of support; the delight with which he showed that hurrying of the precessional motion caused the gyrostat to rise, and retarding the precessional motion caused the gyrostat to fall, so that the freedom to precess was the secret of its not falling; the immediate application of the study of the gyrostat to the explanation of the precession of the equinoxes, and illustration by a model . . . - all these delighted his hearers, and made the lecture memorable.
>
> Andrew Gray, Lord Kelvin, An Account of his Scientific Life and Work

This paragraph by Gray deals with motion, but still it is more concerned

with human responses - Kelvin's obvious pleasure in watching the top, and his student's evident delight in watching both Kelvin and Kelvin's top. At the same time it says much about the top's movement, hints at the reasons behind it, and mentions how understanding the top has led to understanding the precession of the earth's axis in space.

Gray used some of the everyday words for motion: rise, fall, spin, hurry, retard. But he used other words and other phrases, too - more technical, less familiar: precess, center of gravity, equinoxes. Technical words are important for a scientific description of motion. When the scientist has dissected a motion and laid out its components, the need for new terms enters, the need for words with more precise meanings, words not clothed with connotations of emotional response. Still, the scientist never can (and never really wants to), lose the connotations of common words entirely. For example, here is Lord Kelvin's attempt to define precession (see Fig. 1.10), in the sense that Gray used it:

> This we call positive precessional rotation. It is the case of a common spinning-top (peery), spinning on a very fine point which remains at rest in a hollow or hole bored by itself; not sleeping upright, nor nodding, but sweeping its axis round in a circular cone whose axis is vertical.
>
> William Thomson (Lord Kelvin) and P. G. Tait, Treatise on Natural Philosophy

This definition is interesting in several ways. For one thing, it seems strange today that Kelvin, a Scot, should feel the need to explain "spinning-top" by adding "peery," an obscure word to most of us, but one that Kelvin evidently thought more colloquial. Think for a moment of how Kelvin went about his definition. He used the words of boys spinning tops for fun, who then, and still today, say a top sleeps when its axis is nearly straight up, and that it nods as it slows and finally falls. He reminded his readers of something they all had seen and of the everyday words for it. (He obviously assumed that most of his readers once played with tops.) In fact, this is the best way to define new words - to remind the reader of something he knows already and with words he might use himself. Having once given this definition Kelvin never returns to the picture he employed. It is clear, though, that when he wrote, "positive precessional rotation," he brought this image to his own mind, and that he expected his readers to do the same.

Of course, it is not necessary to use as many words as Kelvin did to define precession. Another, more austere, and to some, more scientific definition is this:

> When the axis of the top travels round the vertical making a constant angle i with it, the motion is called steady or precessional.
>
> E. J. Routh, Treatise on the Dynamics of a System of Rigid Bodies

All that refers to direct, human experience is missing here. The top is now just something with an axis, no longer a bright-painted toy spinning on the ground. And it is not the top that moves, but its axis, an imagined line in space, and this line moves about another imagined line, the vertical. There is no poetry here, only geometry. This is an exact, precise, and economical definition, but it is abstract, and incomplete. It does not describe what anyone watching a real top sees. In fact, it is only a few abstractions from the real top's motion on which the physicist-definer has concentrated his attention.

The treatment of speed and acceleration demonstrates the value of simple calculus in analyzing and describing motion.

8 Motion

Richard P. Feynman, Robert B. Leighton and Matthew Sands

A chapter from *The Feynman Lectures on Physics—Volume 1*, 1963.

8–1 Description of motion

In order to find the laws governing the various changes tnat take place in bodies as time goes on, we must be able to *describe* the changes and have some way to record them. The simplest change to observe in a body is the apparent change in its position with time, which we call motion. Let us consider some solid object with a permanent mark, which we shall call a point, that we can observe. We shall discuss the motion of the little marker, which might be the radiator cap of an automobile or the center of a falling ball, and shall try to describe the fact that it moves and how it moves.

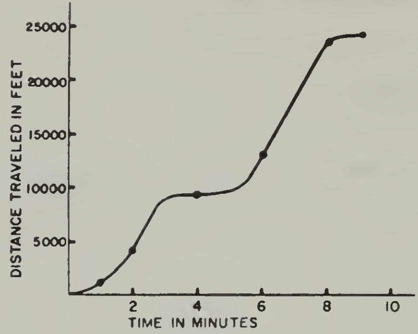
These examples may sound trivial, but many subtleties enter into the description of change. Some changes are more difficult to describe than the motion of a point on a solid object, for example the speed of drift of a cloud that is drifting very slowly, but rapidly forming or evaporating, or the change of a woman's mind. We do not know a simple way to analyze a change of mind, but since the cloud can be represented or described by many molecules, perhaps we can describe the motion of the cloud in principle by describing the motion of all its individual molecules. Likewise, perhaps even the changes in the mind may have a parallel in changes of the atoms inside the brain, but we have no such knowledge yet.

At any rate, that is why we begin with the motion of points; perhaps we should think of them as atoms, but it is probably better to be more rough in the beginning and simply to think of some kind of small objects—small, that is, compared with the distance moved. For instance, in describing the motion of a car that is going a hundred miles, we do not have to distinguish between the front and the back of the car. To be sure, there are slight differences, but for rough purposes we say "the car," and likewise it does not matter that our points are not absolute points; for our present purposes it is not necessary to be extremely precise. Also, while we take a first look at this subject we are going to forget about the three dimensions of the world. We shall just concentrate on moving in one direction, as in a car on one road. We shall return to three dimensions after we see how to describe motion in one dimension. Now, you may say, "This is all some kind of trivia," and indeed it is. How can we describe such a one-dimensional motion—let us say, of a car? Nothing could be simpler. Among many possible ways, one would be the following. To determine the position of the car at different times, we measure its distance from the starting point and record all the observations.

Table 8–1

t (min)	s (ft)
0	0
1	1200
2	4000
3	9000
4	9500
5	9600
6	13000
7	18000
8	23500
9	24000

Fig. 8–1. Graph of distance versus time for the car.



In Table 8–1, s represents the distance of the car, in feet, from the starting point, and t represents the time in minutes. The first line in the table represents zero distance and zero time—the car has not started yet. After one minute it has started and has gone 1200 feet. Then in two minutes, it goes farther—notice that it picked up more distance in the second minute—it has accelerated; but something happened between 3 and 4 and even more so at 5—it stopped at a light perhaps? Then it speeds up again and goes 13,000 feet by the end of 6 minutes, 18,000 feet at the end of 7 minutes, and 23,500 feet in 8 minutes; at 9 minutes it has advanced to only 24,000 feet, because in the last minute it was stopped by a cop.

That is one way to describe the motion. Another way is by means of a graph. If we plot the time horizontally and the distance vertically, we obtain a curve something like that shown in Fig. 8–1. As the time increases, the distance increases, at first very slowly and then more rapidly, and very slowly again for a little while at 4 minutes; then it increases again for a few minutes and finally, at 9 minutes, appears to have stopped increasing. These observations can be made from the graph, without a table. Obviously, for a complete description one would have to know where the car is at the half-minute marks, too, but we suppose that the graph means something, that the car has some position at all the intermediate times.

The motion of a car is complicated. For another example we take something that moves in a simpler manner, following more simple laws: a falling ball. Table 8–2 gives the time in seconds and the distance in feet for a falling body. At zero seconds the ball starts out at zero feet, and at the end of 1 second it has fallen 16 feet. At the end of 2 seconds, it has fallen 64 feet, at the end of 3 seconds, 144 feet, and so on; if the tabulated numbers are plotted, we get the nice parabolic curve shown in Fig. 8–2. The formula for this curve can be written as

$$s = 16t^2. \tag{8.1}$$

This formula enables us to calculate the distances at any time. You might say there ought to be a formula for the first graph too. Actually, one may write such a formula abstractly, as

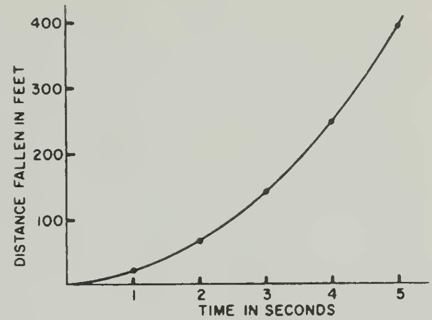
$$s = f(t), \tag{8.2}$$

meaning that s is some quantity depending on t or, in mathematical phraseology,

Table 8–2

t (sec)	s (ft)
0	0
1	16
2	64
3	144
4	256
5	400
6	576

Fig. 8–2. Graph of distance versus time for a falling body.



s is a function of t. Since we do not know what the function is, there is no way we can write it in definite algebraic form.

We have now seen two examples of motion, adequately described with very simple ideas, no subtleties. However, there *are* subtleties—several of them. In the first place, what do we mean by *time* and *space?* It turns out that these deep philosophical questions have to be analyzed very carefully in physics, and this is not so easy to do. The theory of relativity shows that our ideas of space and time are not as simple as one might think at first sight. However, for our present purposes, for the accuracy that we need at first, we need not be very careful about defining things precisely. Perhaps you say, "That's a terrible thing—I learned that in science we have to define *everything* precisely." We cannot define *anything* precisely! If we attempt to, we get into that paralysis of thought that comes to philosophers, who sit opposite each other, one saying to the other, "You don't know what you are talking about!" The second one says, "What do you mean by *know?* What do you mean by *talking?* What do you mean by *you?*," and so on. In order to be able to talk constructively, we just have to agree that we are talking about roughly the same thing. You know as much about time as we need for the present, but remember that there are some subtleties that have to be discussed; we shall discuss them later.

Another subtlety involved, and already mentioned, is that it should be possible to imagine that the moving point we are observing is always located somewhere. (Of course when we are looking at it, there it is, but maybe when we look away it isn't there.) It turns out that in the motion of atoms, that idea also is false—we cannot find a marker on an atom and watch it move. That subtlety we shall have to get around in quantum mechanics. But we are first going to learn what the problems are before introducing the complications, and *then* we shall be in a better position to make corrections, in the light of the more recent knowledge of the subject. We shall, therefore, take a simple point of view about time and space. We know what these concepts are in a rough way, and those who have driven a car know what speed means.

8–2 Speed

Even though we know roughly what "speed" means, there are still some rather deep subtleties; consider that the learned Greeks were never able to adequately describe problems involving velocity. The subtlety comes when we try to compre-

hend exactly what is meant by "speed." The Greeks got very confused about this, and a new branch of mathematics had to be discovered beyond the geometry and algebra of the Greeks, Arabs, and Babylonians. As an illustration of the difficulty, try to solve this problem by sheer algebra: A balloon is being inflated so that the volume of the balloon is increasing at the rate of $100\ \text{cm}^3$ per second; at what speed is the radius increasing when the volume is $1000\ \text{cm}^3$? The Greeks were somewhat confused by such problems, being helped, of course, by some very confusing Greeks. To show that there were difficulties in reasoning about speed at the time, Zeno produced a large number of paradoxes, of which we shall mention one to illustrate his point that there are obvious difficulties in thinking about motion. "Listen," he says, "to the following argument: Achilles runs 10 times as fast as a tortoise, nevertheless he can never catch the tortoise. For, suppose that they start in a race where the tortoise is 100 meters ahead of Achilles; then when Achilles has run the 100 meters to the place where the tortoise was, the tortoise has proceeded 10 meters, having run one-tenth as fast. Now, Achilles has to run another 10 meters to catch up with the tortoise, but on arriving at the end of that run, he finds that the tortoise is still 1 meter ahead of him; running another meter, he finds the tortoise 10 centimeters ahead, and so on, *ad infinitum*. Therefore, at any moment the tortoise is always ahead of Achilles and Achilles can never catch up with the tortoise." What is wrong with that? It is that a finite amount of time can be divided into an infinite number of pieces, just as a length of line can be divided into an infinite number of pieces by dividing repeatedly by two. And so, although there are an infinite number of steps (in the argument) to the point at which Achilles reaches the tortoise, it doesn't mean that there is an infinite amount of *time*. We can see from this example that there are indeed some subtleties in reasoning about speed.

In order to get to the subtleties in a clearer fashion, we remind you of a joke which you surely must have heard. At the point where the lady in the car is caught by a cop, the cop comes up to her and says, "Lady, you were going 60 miles an hour!" She says, "That's impossible, sir, I was travelling for only seven minutes. It is ridiculous—how can I go 60 miles an hour when I wasn't going an hour?" How would you answer her if you were the cop? Of course, if you were really the cop, then no subtleties are involved; it is very simple: you say, "Tell that to the judge!" But let us suppose that we do not have that escape and we make a more honest, intellectual attack on the problem, and try to explain to this lady what we mean by the idea that she was going 60 miles an hour. Just what *do* we mean? We say, "What we mean, lady, is this: if you kept on going the same way as you are going now, in the next hour you would go 60 miles." She could say, "Well, my foot was off the accelerator and the car was slowing down, so if I kept on going that way it would not go 60 miles." Or consider the falling ball and suppose we want to know its speed at the time three seconds if the ball kept on going the way it is going. What does that mean—kept on *accelerating*, going faster? No—kept on going with the same *velocity*. But that is what we are trying to define! For if the ball keeps on going the way it is going, it will just keep on going the way it is going. Thus we need to define the velocity better. What has to be kept the same? The lady can also argue this way: "If I kept on going the way I'm going for one more hour, I would run into that wall at the end of the street!" It is not so easy to say what we mean.

Many physicists think that measurement is the only definition of anything. Obviously, then, we should use the instrument that measures the speed—the

speedometer—and say, "Look, lady, your speedometer reads 60." So she says, "My speedometer is broken and didn't read at all." Does that mean the car is standing still? We believe that there is something to measure before we build the speedometer. Only then can we say, for example, "The speedometer isn't working right," or "the speedometer is broken." That would be a meaningless sentence if the velocity had no meaning independent of the speedometer. So we have in our minds, obviously, an idea that is independent of the speedometer, and the speedometer is meant only to measure this idea. So let us see if we can get a better definition of the idea. We say, "Yes, of course, before you went an hour, you would hit that wall, but if you went one second, you would go 88 feet; lady, you were going 88 feet per second, and if you kept on going, the next second it would be 88 feet, and the wall down there is farther away than that." She says, "Yes, but there's no law against going 88 feet per second! There is only a law against going 60 miles an hour." "But," we reply, "it's the same thing." If it *is* the same thing, it should not be necessary to go into this circumlocution about 88 feet per second. In fact, the falling ball could not keep going the same way even one second because it would be changing speed, and we shall have to define speed somehow.

Now we seem to be getting on the right track; it goes something like this: If the lady kept on going for another 1/1000 of an hour, she would go 1/1000 of 60 miles. In other words, she does not have to keep on going for the whole hour; the point is that *for a moment* she is going at that speed. Now what that means is that if she went just a little bit more in time, the extra distance she goes would be the same as that of a car that goes at a *steady* speed of 60 miles an hour. Perhaps the idea of the 88 feet per second is right; we see how far she went in the last second, divide by 88 feet, and if it comes out 1 the speed was 60 miles an hour. In other words, we can find the speed in this way: We ask, how far do we go in a very short time? We divide that distance by the time, and that gives the speed. But the time should be made as short as possible, the shorter the better, because some change could take place during that time. If we take the time of a falling body as an hour, the idea is ridiculous. If we take it as a second, the result is pretty good for a car, because there is not much change in speed, but not for a falling body; so in order to get the speed more and more accurately, we should take a smaller and smaller time interval. What we should do is take a millionth of a second, and divide that distance by a millionth of a second. The result gives the distance per second, which is what we mean by the velocity, so we can define it that way. That is a successful answer for the lady, or rather, that is the definition that we are going to use.

The foregoing definition involves a new idea, an idea that was not available to the Greeks in a general form. That idea was to take an *infinitesimal distance* and the corresponding *infinitesimal time*, form the ratio, and watch what happens to that ratio as the time that we use gets smaller and smaller and smaller. In other words, take a limit of the distance travelled divided by the time required, as the time taken gets smaller and smaller, *ad infinitum*. This idea was invented by Newton and by Leibnitz, independently, and is the beginning of a new branch of mathematics, called the *differential calculus*. Calculus was invented in order to describe motion, and its first application was to the problem of defining what is meant by going "60 miles an hour."

Let us try to define velocity a little better. Suppose that in a short time, ϵ, the car or other body goes a short distance x; then the velocity, v, is defined as

$$v = x/\epsilon,$$

an approximation that becomes better and better as the ϵ is taken smaller and smaller. If a mathematical expression is desired, we can say that the velocity equals the limit as the ϵ is made to go smaller and smaller in the expression x/ϵ, or

$$v = \lim_{\epsilon \to 0} \frac{x}{\epsilon}. \tag{8.3}$$

We cannot do the same thing with the lady in the car, because the table is incomplete. We know only where she was at intervals of one minute; we can get a rough idea that she was going 5000 ft/min during the 7th minute, but we do not know, at exactly the moment 7 minutes, whether she had been speeding up and the speed was 4900 ft/min at the beginning of the 6th minute, and is now 5100 ft/min, or something else, because we do not have the exact details in between. So only if the table were completed with an infinite number of entries could we really calculate the velocity from such a table. On the other hand, when we have a complete mathematical formula, as in the case of a falling body (Eq. 8.1), then it is possible to calculate the velocity, because we can calculate the position at any time whatsoever.

Let us take as an example the problem of determining the velocity of the falling ball at the particular time 5 seconds. One way to do this is to see from Table 8–2 what it did in the 5th second; it went $400 - 256 = 144$ ft, so it is going 144 ft/sec; however, that is wrong, because the speed is changing; *on the average* it is 144 ft/sec during this interval, but the ball is speeding up and is really going faster than 144 ft/sec. We want to find out *exactly how fast.* The technique involved in this process is the following: We know where the ball was at 5 sec. At 5.1 sec, the distance that it has gone all together is $16(5.1)^2 = 416.16$ ft (see Eq. 8.1). At 5 sec it had already fallen 400 ft; in the last tenth of a second it fell $416.16 - 400 = 16.16$ ft. Since 16.16 ft in 0.1 sec is the same as 161.6 ft/sec, that is the speed more or less, but it is not exactly correct. Is that the speed at 5, or at 5.1, or halfway between at 5.05 sec, or when *is* that the speed? Never mind —the problem was to find the speed *at 5 seconds,* and we do not have exactly that; we have to do a better job. So, we take one-thousandth of a second more than 5 sec, or 5.001 sec, and calculate the total fall as

$$s = 16(5.001)^2 = 16(25.010001) = 400.160016 \text{ ft}.$$

In the last 0.001 sec the ball fell 0.160016 ft, and if we divide this number by 0.001 sec we obtain the speed as 160.016 ft/sec. That is closer, very close, but it is *still not exact.* It should now be evident what we must do to find the speed exactly. To perform the mathematics we state the problem a little more abstractly: to find the velocity at a special time, t_0, which in the original problem was 5 sec. Now the distance at t_0, which we call s_0, is $16t_0^2$, or 400 ft in this case. In order to find the velocity, we ask, "At the time t_0 + (a little bit), or $t_0 + \epsilon$, where is the body?" The new position is $16(t_0 + \epsilon)^2 = 16t_0^2 + 32t_0\epsilon + 16\epsilon^2$. So it is farther along than it was before, because before it was only $16t_0^2$. This distance we shall call s_0 + (a little bit more), or $s_0 + x$ (if x is the extra bit). Now if we subtract the distance at t_0 from the distance at $t_0 + \epsilon$, we get x, the extra distance gone, as $x = 32t_0 \cdot \epsilon + 16\epsilon^2$. Our first approximation to the velocity is

$$v = \frac{x}{\epsilon} = 32t_0 + 16\epsilon. \tag{8.4}$$

The true velocity is the value of this ratio, x/ϵ, when ϵ becomes vanishingly small. In other words, after forming the ratio, we take the limit as ϵ gets smaller and smaller, that is, approaches 0. The equation reduces to,

$$v \text{ (at time } t_0) = 32t_0.$$

In our problem, $t_0 = 5$ sec, so the solution is $v = 32 \times 5 = 160$ ft/sec. A few lines above, where we took ϵ as 0.1 and 0.01 sec successively, the value we got for v was a little more than this, but now we see that the actual velocity is precisely 160 ft/sec.

8–3 Speed as a derivative

The procedure we have just carried out is performed so often in mathematics that for convenience special notations have been assigned to our quantities ϵ and x. In this notation, the ϵ used above becomes Δt and x becomes Δs. This Δt means "an extra bit of t," and carries an implication that it can be made smaller. The prefix Δ is not a multiplier, any more than $\sin\theta$ means $s \cdot i \cdot n \cdot \theta$—it simply defines a time increment, and reminds us of its special character. Δs has an analogous meaning for the distance s. Since Δ is not a factor, it cannot be cancelled in the ratio $\Delta s/\Delta t$ to give s/t, any more than the ratio $\sin\theta/\sin 2\theta$ can be reduced to 1/2 by cancellation. In this notation, velocity is equal to the limit of $\Delta s/\Delta t$ when Δt gets smaller, or

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}. \tag{8.5}$$

This is really the same as our previous expression (8.3) with ϵ and x, but it has the advantage of showing that something is changing, and it keeps track of what is changing.

Incidentally, to a good approximation we have another law, which says that the change in distance of a moving point is the velocity times the time interval, or $\Delta s = v\,\Delta t$. This statement is true only if the velocity is not changing during that time interval, and this condition is true only in the limit as Δt goes to 0. Physicists like to write it $ds = v\,dt$, because by dt they mean Δt in circumstances in which it is very small; with this understanding, the expression is valid to a close approximation. If Δt is too long, the velocity might change during the interval, and the approximation would become less accurate. For a time dt, approaching zero, $ds = v\,dt$ precisely. In this notation we can write (8.5) as

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}.$$

The quantity ds/dt which we found above is called the "derivative of s with respect to t" (this language helps to keep track of what was changed), and the complicated process of finding it is called finding a derivative, or differentiating. The ds's and dt's which appear separately are called *differentials*. To familiarize you with the words, we say we found the derivative of the function $16t^2$, or the derivative (with respect to t) of $16t^2$ is $32t$. When we get used to the words, the ideas are more easily understood. For practice, let us find the derivative of a more complicated function. We shall consider the formula $s = At^3 + Bt + C$, which

might describe the motion of a point. The letters A, B, and C represent constant numbers, as in the familiar general form of a quadratic equation. Starting from the formula for the motion, we wish to find the velocity at any time. To find the velocity in the more elegant manner, we change t to $t + \Delta t$ and note that s is then changed to $s +$ some Δs; then we find the Δs in terms of Δt. That is to say,

$$\begin{aligned} s + \Delta s &= A(t + \Delta t)^3 + B(t + \Delta t) + C \\ &= At^3 + Bt + C + 3At^2 \Delta t + B \Delta t + 3At(\Delta t)^2 + A(\Delta t)^3, \end{aligned}$$

but since

$$s = At^3 + Bt + C,$$

we find that

$$\Delta s = 3At^2 \Delta t + B \Delta t + 3At(\Delta t)^2 + A(\Delta t)^3.$$

But we do not want Δs—we want Δs divided by Δt. We divide the preceding equation by Δt, getting

$$\frac{\Delta s}{\Delta t} = 3At^2 + B + 3At(\Delta t) + A(\Delta t)^2.$$

As Δt goes toward 0 the limit of $\Delta s/\Delta t$ is ds/dt and is equal to

$$\frac{ds}{dt} = 3At^2 + B.$$

This is the fundamental process of calculus, differentiating functions. The process is even more simple than it appears. Observe that when these expansions contain any term with a square or a cube or any higher power of Δt, such terms may be dropped at once, since they will go to 0 when the limit is taken. After a little practice the process gets easier because one knows what to leave out. There are many rules or formulas for differentiating various types of functions. These can be memorized, or can be found in tables. A short list is found in Table 8–3.

Table 8–3. A Short Table of Derivatives

s, u, v, w are arbitrary functions of t; a, b, c, and n are arbitrary constants

Function	Derivative
$s = t^n$	$\frac{ds}{dt} = nt^{n-1}$
$s = cu$	$\frac{ds}{dt} = c\frac{du}{dt}$
$s = u + v + w + \cdots$	$\frac{ds}{dt} = \frac{du}{dt} + \frac{dv}{dt} + \frac{dw}{dt} + \cdots$
$s = c$	$\frac{ds}{dt} = 0$
$s = u^a v^b w^c \ldots$	$\frac{ds}{dt} = s\left(\frac{a}{u}\frac{du}{dt} + \frac{b}{v}\frac{dv}{dt} + \frac{c}{w}\frac{dw}{dt} + \cdots\right)$

Table 8–4

Velocity of a Falling Ball

t (sec)	v (ft/sec)
0	0
1	32
2	64
3	96
4	128

8–4 Distance as an integral

Now we have to discuss the inverse problem. Suppose that instead of a table of distances, we have a table of speeds at different times, starting from zero. For the falling ball, such speeds and times are shown in Table 8–4. A similar table could be constructed for the velocity of the car, by recording the speedometer reading every minute or half-minute. If we know how fast the car is going at any time, can we determine how far it goes? This problem is just the inverse of the one solved above; we are given the velocity and asked to find the distance. How can we find the distance if we know the speed? If the speed of the car is not constant, and the lady goes sixty miles an hour for a moment, then slows down, speeds up, and so on, how can we determine how far she has gone? That is easy. We use the same idea, and express the distance in terms of infinitesimals. Let us say, "In the first second her speed was such and such, and from the formula $\Delta s = v\,\Delta t$ we can calculate how far the car went the first second at that speed." Now in the next second her speed is nearly the same, but slightly different; we can calculate how far she went in the next second by taking the new speed times the time. We proceed similarly for each second, to the end of the run. We now have a number of little distances, and the total distance will be the sum of all these little pieces. That is, the distance will be the sum of the velocities times the times, or $s = \sum v\,\Delta t$, where the Greek letter $\sum$ (sigma) is used to denote addition. To be more precise, it is the sum of the velocity at a certain time, let us say the i-th time, multiplied by Δt.

$$s = \sum_i v(t_i)\,\Delta t. \tag{8.6}$$

The rule for the times is that $t_{i+1} = t_i + \Delta t$. However, the distance we obtain by this method will not be correct, because the velocity changes during the time interval Δt. If we take the times short enough, the sum is precise, so we take them smaller and smaller until we obtain the desired accuracy. The true s is

$$s = \lim_{\Delta t \to 0} \sum_i v(t_i)\,\Delta t. \tag{8.7}$$

The mathematicians have invented a symbol for this limit, analogous to the symbol for the differential. The Δ turns into a d to remind us that the time is as small as it can be; the velocity is then called v at the time t, and the addition is written as a sum with a great "s," $\int$ (from the Latin *summa*), which has become distorted and is now unfortunately just called an integral sign. Thus we write

$$s = \int v(t)\,dt. \tag{8.8}$$

This process of adding all these terms together is called integration, and it is the opposite process to differentiation. The derivative of this integral is v, so one operator (d) undoes the other ($\int$). One can get formulas for integrals by taking the formulas for derivatives and running them backwards, because they are related to each other inversely. Thus one can work out his own table of integrals by differentiating all sorts of functions. For every formula with a differential, we get an integral formula if we turn it around.

Every function can be differentiated analytically, i.e., the process can be carried out algebraically, and leads to a definite function. But it is not possible in a simple manner to write an analytical value for any integral at will. You can calculate it, for instance, by doing the above sum, and then doing it again with a finer interval Δt and again with a finer interval until you have it nearly right. In general, given some particular function, it is not possible to find, analytically, what the integral is. One may always try to find a function which, when differentiated, gives some desired function; but one may not find it, and it may not exist, in the sense of being expressible in terms of functions that have already been given names.

8–5 Acceleration

The next step in developing the equations of motion is to introduce another idea which goes beyond the concept of velocity to that of *change* of velocity, and we now ask, "How does the velocity *change?*" In previous chapters we have discussed cases in which forces produce changes in velocity. You may have heard with great excitement about some car that can get from rest to 60 miles an hour in ten seconds flat. From such a performance we can see how fast the speed changes, but only on the average. What we shall now discuss is the next level of complexity, which is how fast the velocity is changing. In other words, by how many feet per second does the velocity change in a second, that is, how many feet per second, per second? We previously derived the formula for the velocity of a falling body as $v = 32t$, which is charted in Table 8–4, and now we want to find out how much the velocity changes per second; this quantity is called the acceleration.

Acceleration is defined as the time rate of change of velocity. From the preceding discussion we know enough already to write the acceleration as the derivative dv/dt, in the same way that the velocity is the derivative of the distance. If we now differentiate the formula $v = 32t$ we obtain, for a falling body,

$$a = \frac{dv}{dt} = 32. \tag{8.9}$$

[To differentiate the term $32t$ we can utilize the result obtained in a previous problem, where we found that the derivative of Bt is simply B (a constant). So by letting $B = 32$, we have at once that the derivative of $32t$ is 32.] This means that the velocity of a falling body is changing by 32 feet per second, per second always. We also see from Table 8–4 that the velocity increases by 32 ft/sec in each second. This is a very simple case, for accelerations are usually not constant. The reason the acceleration is constant here is that the force on the falling body is constant, and Newton's law says that the acceleration is proportional to the force.

As a further example, let us find the acceleration in the problem we have already solved for the velocity. Starting with

$$s = At^3 + Bt + C$$

we obtained, for $v = ds/dt$,

$$v = 3At^2 + B.$$

Since acceleration is the derivative of the velocity with respect to the time, we need to differentiate the last expression above. Recall the rule that the derivative of the two terms on the right equals the sum of the derivatives of the individual terms. To differentiate the first of these terms, instead of going through the fundamental process again we note that we have already differentiated a quadratic term when we differentiated $16t^2$, and the effect was to double the numerical coefficient and change the t^2 to t; let us assume that the same thing will happen this time, and you can check the result yourself. The derivative of $3At^2$ will then be $6At$. Next we differentiate B, a constant term; but by a rule stated previously, the derivative of B is zero; hence this term contributes nothing to the acceleration. The final result, therefore, is $a = dv/dt = 6At$.

For reference, we state two very useful formulas, which can be obtained by integration. If a body starts from rest and moves with a constant acceleration, g, its velocity v at any time t is given by

$$v = gt.$$

The distance it covers in the same time is

$$s = \tfrac{1}{2}gt^2.$$

Various mathematical notations are used in writing derivatives. Since velocity is ds/dt and acceleration is the time derivative of the velocity, we can also write

$$a = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2s}{dt^2}, \tag{8.10}$$

which are common ways of writing a second derivative.

We have another law that the velocity is equal to the integral of the acceleration. This is just the opposite of $a = dv/dt$; we have already seen that distance is the integral of the velocity, so distance can be found by twice integrating the acceleration.

In the foregoing discussion the motion was in only one dimension, and space permits only a brief discussion of motion in three dimensions. Consider a particle P which moves in three dimensions in any manner whatsoever. At the beginning of this chapter, we opened our discussion of the one-dimensional case of a moving car by observing the distance of the car from its starting point at various times. We then discussed velocity in terms of changes of these distances with time, and acceleration in terms of changes in velocity. We can treat three-dimensional motion analogously. It will be simpler to illustrate the motion on a two-dimensional diagram, and then extend the ideas to three dimensions. We establish a pair of axes at right angles to each other, and determine the position of the particle at any moment by measuring how far it is from each of the two axes. Thus each position is given in terms of an x-distance and a y-distance, and the motion can be described by constructing a table in which both these distances are given as functions of time.

(Extension of this process to three dimensions requires only another axis, at right angles to the first two, and measuring a third distance, the z-distance. The distances are now measured from coordinate *planes* instead of lines.) Having constructed a table with x- and y-distances, how can we determine the velocity? We first find the components of velocity in each direction. The horizontal part of the velocity, or x-component, is the derivative of the x-distance with respect to the time, or

$$v_x = dx/dt. \tag{8.11}$$

Similarly, the vertical part of the velocity, or y-component, is

$$v_y = dy/dt. \tag{8.12}$$

In the third dimension,

$$v_z = dz/dt. \tag{8.13}$$

Now, given the components of velocity, how can we find the velocity along the actual path of motion? In the two-dimensional case, consider two successive positions of the particle, separated by a short distance Δs and a short time interval $t_2 - t_1 = \Delta t$. In the time Δt the particle moves horizontally a distance $\Delta x \sim v_x \Delta t$, and vertically a distance $\Delta y \sim v_y \Delta t$. (The symbol "$\sim$" is read "is approximately.") The actual distance moved is approximately

$$\Delta s \sim \sqrt{(\Delta x)^2 + (\Delta y)^2}, \tag{8.14}$$

as shown in Fig. 8–3. The approximate velocity during this interval can be obtained by dividing by Δt and by letting Δt go to 0, as at the beginning of the chapter. We then get the velocity as

$$v = \frac{ds}{dt} = \sqrt{(dx/dt)^2 + (dy/dt^2)} = \sqrt{v_x^2 + v_y^2} \tag{8.15}$$

For three dimensions the result is

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}. \tag{8.16}$$

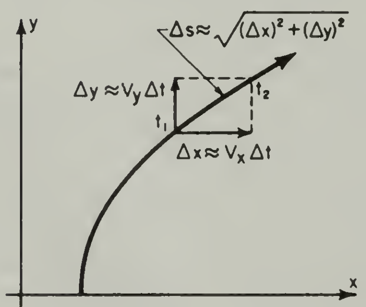


Fig. 8–3. Description of the motion of a body in two dimensions and the computation of its velocity.

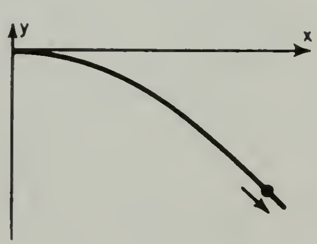


Fig. 8–4. The parabola described by a falling body with an initial horizontal velocity.

In the same way as we defined velocities, we can define accelerations: we have an x-component of acceleration a_x, which is the derivative of v_x, the x-component of the velocity (that is, $a_x = d^2x/dt^2$, the second derivative of x with respect to t), and so on.

Let us consider one nice example of compound motion in a plane. We shall take a motion in which a ball moves horizontally with a constant velocity u, and at the same time goes vertically downward with a constant acceleration $-g$; what is the motion? We can say $dx/dt = v_x = u$. Since the velocity v_x is constant,

$$x = ut, \tag{8.17}$$

and since the downward acceleration $-g$ is constant, the distance y the object falls can be written as

$$y = -\tfrac{1}{2}gt^2. \tag{8.18}$$

What is the curve of its path, i.e., what is the relation between y and x? We can eliminate t from Eq. (8.18), since $t = x/u$. When we make this substitution we find that

$$y = -\frac{g}{2u^2}x^2. \tag{8.19}$$

This relation between y and x may be considered as the equation of the path of the moving ball. When this equation is plotted we obtain a curve that is called a parabola; any freely falling body that is shot out in any direction will travel in a parabola, as shown in Fig. 8–4.

The twentieth century artist has been able to exploit his interest in motion in various ways in works of art.

9 Representation of Movement

Gyorgy Kepes

A chapter from his book *Language of Vision,* 1944.

Matter, the physical basis of all spatial experience and thus the source material of representation, is kinetic in its very essence. From atomic happenings to cosmic actions, all elements in nature are in perpetual interaction—in a flux complete. We are living a mobile existence. The earth is rotating; the sun is moving; trees are growing; flowers are opening and closing; clouds are merging, dissolving, coming and going; light and shadow are hunting each other in an indefatigable play; forms are appearing and disappearing; and man, who is experiencing all this, is himself subject to all kinetic change. The perception of physical reality cannot escape the quality of movement. The very understanding of spatial facts, the meaning of extension or distances, involves the notion of time—a fusion of space-time which is movement. "Nobody has ever noticed a place except at a time or a time except at a place," said Minkowsky in his *Principles of Relativity.*

The sources of movement perception

As in a wild jungle one cuts new paths in order to progress further, man builds roads of perception on which he is able to approach the mobile world, to discover order in its relationships. To build these avenues of perceptual grasp he relies on certain natural factors. One is the nature of the retina, the sensitive surface on which the mobile panorama is projected. The second is the sense of movement of his body—the kinesthetic sensations of his eye muscles, limbs, head, which have a direct correspondence with the happenings around him. The third is the memory association of past experience, visual and non-visual; his knowledge about the laws of the physical nature of the surrounding object-world.

The shift of the retinal image

We perceive any successive stimulation of the retinal receptors as movement, because such progressive stimulations are in dynamic interaction with fixed stimulations, and therefore the two different types of stimulation can be perceived in a unified whole only as a dynamic process, movement. If the retina is stimulated with stationary impacts that follow one another

in rapid succession, the same sensation of optical movement is induced. Advertising displays with their rapidly flashing electric bulbs are perceived in continuity through the persistence of vision and therefore produce the sensation of movement, although the spatial position of the light bulbs is stationary. The movement in the motion picture is based upon the same source of the visual perception.

The changes of any optical data indicating spatial relationships, such as size, shape, direction, interval, brightness, clearness, color, imply motion. If the retinal image of any of these signs undergoes continuous regular change, expansion or contraction, progression or graduation, one perceives an approaching or receding, expanding or contracting movement. If one sees a growing or disappearing distance between these signs, he perceives a horizontal or vertical movement.

"Suppose for instance, that a person is standing still in a thick woods, where it is impossible for him to distinguish, except vaguely and roughly in a mass of foliage and branches all around him, what belongs to one tree and what to another, and how far the trees are separated. The moment he begins to move forward, however, everything disentangles itself and immediately he gets an apperception of the content of the woods and the relationships of objects to each other in space."•

From a moving train, the closer the object the faster it seems to move. A far-away object moves slowly and one very remote appears to be stationary. The same phenomenon, with a lower relative velocity, may be noticed in walking, and with a still higher velocity in a landing aeroplane or in a moving elevator.

The role of relative velocity

The velocity of motion has an important conditioning effect. Motion can be too fast or too slow to be perceived as such by our limited sensory receiving set. The growth of trees or of man, the opening of flowers, the evaporation of water are movements beyond the threshold of ordinary visual grasp. One does not see the movement of the hand of a watch, of a ship on a distant horizon. An aeroplane in the highest sky seems to hang motionless. No one can see the traveling of light as such. In certain less rapid motions beyond the visual grasp, one is able, however, to observe the optical transformation of movement into the illusion of a solid. A rapidly whirled torch loses its characteristic physical extension, but it submerges into another three-dimensional-appearing solid—into the virtual volume of a cone or a sphere. Our inability to distinguish sharply beyond a certain interval of optical impacts makes the visual impressions a blur which serves as a bridge to a new optical form. The degree of velocity of its movement will determine the apparent density of that new form. The optical density of the visible world is in a great degree conditioned by our visual ability, which has its particular limitations.

• *Helmholtz, Physiological Optics*

The kinesthetic sensation

When a moving object comes into the visual field, one pursues it by a corresponding movement of his eyes, keeping it in a stationary or nearly stationary position on the retina. Retinal stimulation, then, cannot alone account for the sensation of movement. Movement-experience, which is undeniably present in such a case, is induced by the sensation of muscle movements. Each individual muscle-fibre contains a nerve end, which registers every movement the muscle makes. That we are able to sense space in the dark, evaluate direction-distances in the absence of contacted bodies, is due to this muscular sensation—the kinesthetic sensation.

E. G. Lukacs. *Action*
From Herbert Bayer Design Class

H. L. Carpenter. *Movement* •

• *Work done for the author's course in Visual Fundamentals.*

Paul Rand. *Cover Design*

Memory sources

Experience teaches man to distinguish things and to evaluate their physical properties. He knows that bodies have weight; unsupported they will of necessity fall. When, therefore, he sees in midair a body he knows to be heavy, he automatically associates the direction and velocity of its downward course. One is also accustomed to seeing small objects as more mobile than large ones. A man is more mobile than a mountain; a bird is more frequently in motion than a tree, the sky, or other visible units in its background. Everything that one experiences is perceived in a polar unity in which one pole is accepted as a stationary background and the other as a mobile, changing figure.

Through all history painters have tried to suggest movement on the stationary picture surface, to translate some of the optical signs of movement-experience into terms of the picture-image. Their efforts, however, have been isolated attempts in which one or the other sources of movement-experience were drawn upon; the shift of the retinal image, the

kinesthetic experience, or the memory of past experiences were suggested in two-dimensional terms.

These attempts were conditioned mainly by the habit of using things as the basic measuring unit for every event in nature. The constant characteristics of the things and objects, first of all the human body, animals, sun, moon, clouds, or trees, were used as the first fixed points of reference in seeking relationships in the optical turmoil of happenings.

Therefore, painters tried first to represent motion by suggesting the visible modifications of objects in movement. They knew the visual characteristics of stationary objects and therefore every observable change served to suggest movement. The prehistoric artist knew his animals, knew, for example, how many legs they had. But when he saw an animal in really speedy movement, he could not escape seeing the visual modification of the known spatial characteristics. The painter of the Altamiro caves who pictures a running reindeer with numerous legs, or the twentieth century cartoonist picturing a moving face with many superimposed profiles, is stating a relationship between what he knows and what he sees.

Ch. D. Gibson.
The Gentleman's Dilemma cc. 1900

Other painters, seeking to indicate movement, utilized the expressive distortion of the moving bodies. Michaelangelo, Goya, and also Tintoretto, by elongating and stretching the figure, showed distortion of the face under the expression of strains of action and mobilized numerous other psychological references to suggest action.

The smallest movement is more possessive of the attention than the greatest wealth of relatively stationary objects. Painters of many different periods observed this well and explored it creatively. The optical vitality of the moving units they emphasized by dynamic outlines, by a vehement interplav of vigorous contrast of light and dark, and by extreme contrast of colors. In various paintings of Tintoretto, Maffei, Veronese, and Goya, the optical wealth and intensity of the moving figures are juxtaposed against the submissive, neutral, visual pattern of the stationary background.

The creative exploitation of the successive stimulations of the retinal receptors in terms of the picture surface was another device many painters found useful. Linear continuance arrests the attention and forces the eye into a pursuit movement. The eye, following the line, acts as if it were on the path of a moving thing and attributes to the line the quality of movement. When the Greek sculptors organized the drapery of their figures which they represented in motion, the lines were conceived as optical forces making the eye pursue their direction.

We know that a heavy object in a background that does not offer substantial resistance will fall. Seeing such an object we interpret it as action.

Harunobu. *Windy Day Under Willow*
Courtesy of The Art Institute of Chicago

Maffei. *Painting*

We make a kind of psychological qualification. Every object seen and interpreted in a frame of reference of gravitation is endowed with potential action and could appear as falling, rolling, moving. Because we customarily assume an identity between the horizontal and vertical directions on the picture surface and the main directions of space as we perceive them in our everyday experiences, every placing of an object representation on the picture surface which contradicts the center of gravity, the main direction of space—the horizontal or vertical axis—causes that object to appear to be in action. Top and bottom of the picture surface have a significance in this respect.

Whereas the visual representation of depth had found various complete systems, such as linear perspective, modelling by shading, a parallel development had never taken place in the visual representation of motion. Possibly this has been because the tempo of life was comparatively slow; therefore, the ordering and representation of events could be compressed without serious repercussions in static formulations. Events were measured by things, static forms identical with themselves, in a perpetual fixity. But this static point of view lost all semblance of validity when daily experiences bombarded man with a velocity of visual impacts in which the fixity of the things, their self-identity, seemed to melt away.

G. McVicker. ***Study of Linear Movement***

Work done for the author's course in Visual Fundamentals

Sponsored by The Art Director's Club of Chicago. 1938

Lee King. ***Study of Movement Representation***

Work done for the author's course in Visual Fundamentals

School of Design in Chicago

The more complex life became, the more dynamic relationships confronted man, in general and in particular, as visual experiences, the more necessary it became to revaluate the old relative conceptions about the fixity of things and to look for a new way of seeing that could interpret man's surroundings in their change. It was no accident that our age made the first serious search for a reformulation of the events in nature into dynamic terms. This reformulation of our ideas about the world included almost all the aspects one perceives. The interpretation of the objective world in the terms of physics, the understanding of the living organism, the reading of the inner movement of social processes, and the visual interpretation of events were, and still are, struggling for a new gauge elastic enough to expand and contract in following the dynamic changes of events.

The influence of the technological conditions

The environment of the man living today has a complexity which cannot be compared with any environment of any previous age. The skyscrapers, the street with its kaleidoscopic vibration of colors, the window-displays with their multiple mirroring images, the street cars and motor cars, produce a dynamic simultaneity of visual impression which cannot be perceived in the terms of inherited visual habits. In this optical turmoil the fixed objects appear utterly insufficient as the measuring tape of the events. The artificial light, the flashing of electric bulbs, and the mobile game of the many new types of light-sources bombard man with kinetic color sensations having a keyboard never before experienced. Man, the spectator, is himself more mobile than ever before. He rides in streetcars, motorcars and aeroplanes and his own motion gives to optical impacts a tempo far beyond the threshold of a clear object-perception. The machine man operates adds its own demand for a new way of seeing. The complicated interactions of its mechanical parts cannot be conceived in a static way; they must be perceived by understanding of their movements. The motion picture, television, and, in a great degree, the radio, require a new thinking, i.e., seeing, that takes into account qualities of change, interpenetration and simultaneity.
Man can face with success this intricate pattern of the optical events only as he can develop a speed in his perception to match the speed of his environment. He can act with confidence only as he learns to orient himself in the new mobile landscape. He needs to be quicker than the event he intends to master. The origin of the word "speed" has a revealing meaning. In original form in most languages, speed is intimately connected with success. Space and speed are, moreover, in some early forms of languages, interchangeable in meaning. Orientation, which is the basis of survival, is guaranteed by the speed of grasping the relationships of the events with which man is confronted.

Social and psychological motivations

Significantly, the contemporary attempts to represent movement were made in the countries where the vitality of living was most handicapped by outworn social conditions. In Italy, technological advances and their economical-social consequences, were tied with the relics of past ideas, institutions. The advocates of change could see no clear, positive direction. Change as they conceived it meant expansion, imperialist power policy. The advance guard of the expanding imperialism identified the past with the monuments of the past, and with the keepers of these monuments; and they tried to break, with an uninhibited vandalism, everything which seemed to them to fetter the progress toward their goals. "We want to free our country from the fetid gangrene of professors, archaeologists, guides and antique shops," proclaimed the futurist manifesto of 1909. The violence of imperialist expansion was identified with vitality; with the flux of life itself. Everything which stood in the way of this desire of the beast to reach his prey was to be destroyed. Movement, speed, velocity became their idols. Destructive mechanical implements, the armoured train, machine gun, a blasting bomb, the aeroplane, the motor car, boxing, were adored symbols of the new virility they sought.

In Russia, where the present was also tied to the past and the people were struggling for the fresh air of action, interest also focused on the dynamic qualities of experience. The basic motivation of reorientation toward a kinetic expression there was quite similar to that of the Italian futurists. It was utter disgust with a present held captive by the past. Russia's painters, writers, like Russia's masses, longed to escape into a future free from the ties of outworn institutions and habits. Museums, grammar, authority, were conceived of as enemies; force, moving masses, moving machines were friends. But this revolt against stagnant traditions, this savage ridiculing of all outworn forms, opened the way for the building of a broader world. The old language, which as Mayakovsky said "was too feeble to catch up with life," was reorganized into kinetic idioms of revolutionary propaganda. The visual language of the past, from whose masters Mayakovsky asked with just scorn, "Painters will you try to capture speedy cavalry with the tiny net of contours?" was infused with new living blood of motion picture vision.

In their search to find an optical projection which conformed to the dynamic reality as they sensed and comprehended it, painters unconsciously repeated the path traced by advancing physical science.

Their first step was to represent on the same picture-plane a sequence of positions of a moving body. This was basically nothing but a cataloging of stationary spatial locations. The idea corresponded to the concept of classical physics, which describes objects existing in three-dimensional space and changing locations in sequence of absolute time. The concept of the object was kept. The sequence of events frozen on the picture-

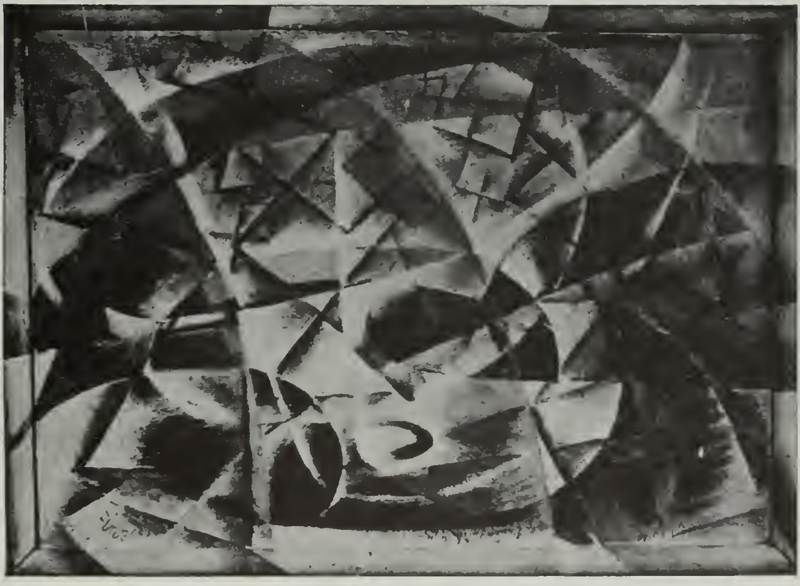
plane only amplified the contradiction between the dynamic reality and the fixity of the three-dimensional object-concept.

Their second step was to fuse the different positions of the object by filling out the pathway of their movement. Objects were no longer considered as isolated, fixed units. Potential and kinetic energies were included as optical characteristics. The object was regarded to be either in active motion, indicating its direction by "lines of force," or in potential motion, pregnant with lines of force, which pointed the direction in which the object would go if freed. The painters thus sought to picture the mechanical point of view of nature, devising optical equivalents for mass, force, and gravitation. This innovation signified important progress, because the indicated lines of forces could function as the plastic forces of two-dimensional picture-plane.

The third step was guided by desire to integrate the increasingly complicated maze of movement-directions. The chaotic jumble of centrifugal line of forces needed to be unified. Simultaneous representation of the numerous visible aspects composing an event was the new representational technique here introduced. The cubist space analysis was synchronized with the line of forces. The body of the moving object, the path of its movement and its background were portrayed in the same picture by fusing all these elements in a kinetic pattern. The romantic language of the futurist manifestos describes the method thus: "The simultaneosity of soul in a work of art; such is the exciting aim of our art. In painting a figure on a balcony, seen from within doors, we shall not confine the view to what can be seen through the frame of the window; we shall give the sum total of the visual sensation of the street, the double row of houses extending right and left the flowered balconies, etc. . . . in other words, a simultaneity of environment and therefore a dismemberment and dislocation of objects, a scattering and confusion of details independent of one and another and without reference to accepted logic," said Marinetti. This concept shows a great similarity to the idea expressed by Einstein, expounding as a physicist the space-time interpretation of the general theory of relativity. "The world of events can be described by a static picture thrown onto the background of the four dimensional time-space continuum. In the past science described motion as happenings in time, general theory of relativity interprets events existing in space-time."

The closest approximation to representation of motion in the genuine terms of the picture-plane was achieved by the utilization of color planes as the organizing factor. The origin of color is light, and colors on the picture surface have an intrinsic tendency to return to their origin. Motion, therefore, is inherent in color. Painters intent on realizing the full motion potentialities of color believed that the image becomes a form only in the progressive interrelationships of opposing colors. Adjacent color-surfaces exhibit contrast effects. They reinforce each other in hue, saturation, and intensity.

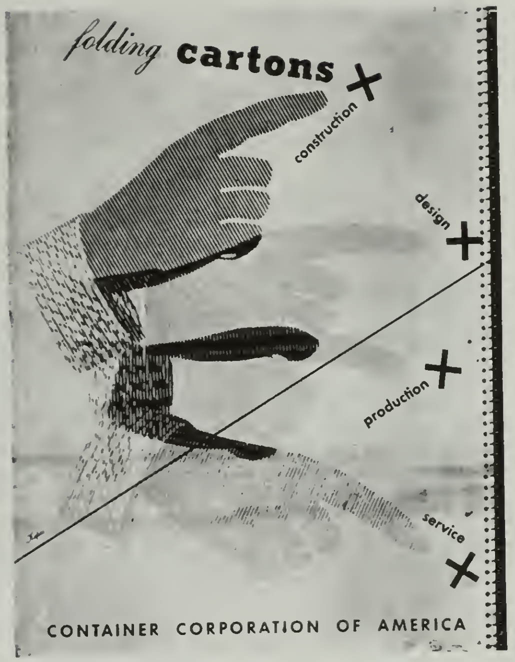
Giacomo Balla. ***Dog on Leash 1912.*** *Courtesy of The Museum of Modern Art*



Giacomo Balla. ***Automobile and Noise.*** *Courtesy of Art of This Century*

Marcel Duchamp. ***Sad Young Man in a Train.***
Courtesy of Art of This Century

Marcel Duchamp.
Nude Descending the Stairs 1912
Reproduction Courtesy
The Art Institute of Chicago

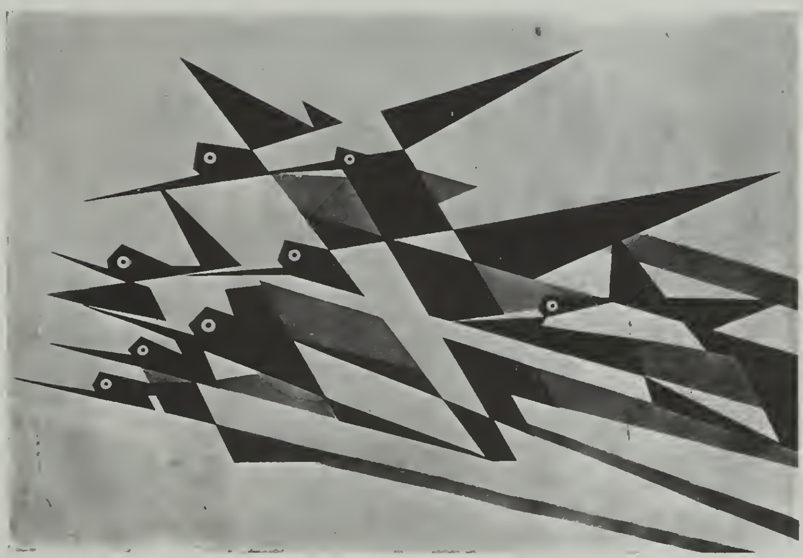


Gyorgy Kepes. ***Advertising Design 1938***
Courtesy of Container Corporation of America

Herbert Matter. ***Advertising Design***
Courtesy of Container Corporation of America

Harold E. Edgerton. *Golfer*

Soviet Poster



E. McKnight Kauffer. ***The Early Bird 1919***
Courtesy of The Museum of Modern Art

Delauney. ***Circular Rhythm*** *Courtesy of The Guggenheim Museum of Non-Objective Art*

The greater the intensity of the color-surfaces achieved by a carefully organized use of simultaneous and successive contrast, the greater their spatial movement color in regard to picture-plane. Their advancing, receding, contracting and circulating movement on the surface creates a rich variety, circular, spiral, pendular, etc., in the process of moulding them into one form which is light or, in practical terms, grey. "Form is movement," declared Delaunay. The classical continuous outline of the objects was therefore eliminated and a rhythmic discontinuity created by grouping colors in the greatest possible contrast. The picture-plane, divided into a number of contrasting color-surfaces of different hue, saturation, and intensity, could be perceived only as a form, as a unified whole in the dynamic sequence of visual perception. The animation of the image they achieved is based upon the progressive steps in bringing opposing colors into balance.

The centrifugal and centripetal forces of the contrasting color-planes move forward and backward, up and down, left and right, compelling the spectator to a kinetic participation as he follows the intrinsic spatial-direction of colors. The dynamic quality is based upon the genuine movement of plastic forces in their tendency toward balance. Like a spinning top or the running wheel of a bicycle, which can find its balance only in movement, the plastic image achieves unity in movement, in perpetual relations of contrasting colors.

A. M. Cassandre. *Poster*

In his witty and provocative book, *About Vectors,* from which this opening chapter is taken, Banesh Hoffmann confesses that he seeks here "to instruct primarily by being disturbing and annoying."

10 Introducing Vectors

Banesh Hoffmann

A chapter from his book *About Vectors,* 1966.

Making good definitions is not easy. The story goes that when the philosopher Plato defined *Man* as "a two-legged animal without feathers," Diogenes produced a plucked cock and said "Here is Plato's man." Because of this, the definition was patched up by adding the phrase "and having broad nails"; and there, unfortunately, the story ends. But what if Diogenes had countered by presenting Plato with the feathers he had plucked?

Exercise 1.1 What? [Note that Plato would now have feathers.]

Exercise 1.2 Under what circumstances could an elephant qualify as a man according to the above definition?

A *vector* is often defined as *an entity having both magnitude and direction.* But that is not a good definition. For example, an arrow-headed line segment like this

has both magnitude (its length) and direction, and it is often used as a drawing of a vector; yet it is not a vector. Nor is an archer's arrow a vector, though it, too, has both magnitude and direction.

To define a vector we have to add to the above definition something analogous to "and having broad nails," and even then we shall find ourselves not wholly satisfied with the definition. But it will let us start, and we can try patching up the definition further as we proceed—and we may even find ourselves replacing it by a quite different sort of definition later on. If, in the end, we have the uneasy feeling that we have still not found a completely satisfactory definition of a vector, we need not be dismayed, for it is the nature of definitions not to be completely satisfactory, and we shall have learned pretty well what a vector is anyway, just as we know, without being able to give a satisfactory definition, what a man is—well enough to be able to criticize Plato's definition.

Exercise 1.3 Define a *door.*

Exercise 1.4 Pick holes in your definition of a *door.*

Exercise 1.5 According to your definition, is a movable partition between two rooms a door?

2. THE PARALLELOGRAM LAW

The main thing we have to add to the magnitude-and-direction definition of a vector is the following:

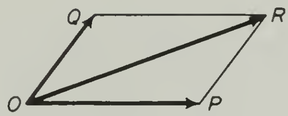


Figure 2.1

Let us think of vectors as having definite locations. And let the arrow-headed line segments $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ in Figure 2.1 represent the magnitudes, directions, and locations of two vectors starting at a common point O. Complete the parallelogram formed by $\overrightarrow{OP}$ and $\overrightarrow{OQ}$, and draw the diagonal OR. Then, when taken together, the two vectors represented by $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ are equivalent to a single vector represented by the arrow-headed line segment $\overrightarrow{OR}$. This vector is called the *resultant* of the vectors represented by $\overrightarrow{OP}$ and $\overrightarrow{OQ}$, and the above crucial property of vectors is called the *parallelogram law* of combination of vectors.

Exercise 2.1 Find (*a*) by drawing and measurement, and (*b*) by calculation using Pythagoras' theorem, the magnitude and direction of the resultant of two vectors $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ if each has magnitude 3, and $\overrightarrow{OP}$ points thus $\longrightarrow$ while $\overrightarrow{OQ}$ points perpendicularly, thus $\uparrow$. [*Ans.* The magnitude is $3\sqrt{2}$, or approximately 4.2, and the direction bisects the right angle between $\overrightarrow{OP}$ and $\overrightarrow{OQ}$.]

Exercise 2.2 Show that the resultant of two vectors $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ that point in the same direction is a vector pointing in the same direction and having a magnitude equal to the sum of the magnitudes of $\overrightarrow{OP}$ and $\overrightarrow{OQ}$. [Imagine the parallelogram in Figure 2.1 squashed flat into a line.]

Exercise 2.3 Taking a hint from Exercise 2.2, describe the resultant of two vectors $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ that point in opposite directions.

Exercise 2.4 In Exercise 2.3, what would be the resultant if $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ had equal magnitudes? [Do you notice anything queer when you compare this resultant vector with the definition of a vector?]

Exercise 2.5 Observe that the resultant of $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ is the same as the resultant of $\overrightarrow{OQ}$ and $\overrightarrow{OP}$. [This is trivially obvious, but keep it in mind nevertheless. We shall return to it later.]

In practice, all we need to draw is half the parallelogram in Figure 2.1—either triangle OPR or triangle OQR. When we do this it looks as if we had combined two vectors $\overrightarrow{OP}$ and $\overrightarrow{PR}$ (or $\overrightarrow{OQ}$ and $\overrightarrow{QR}$) end-to-end like this, even

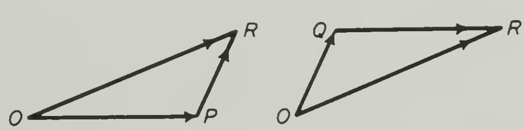
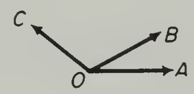


Figure 2.2 (For clarity, the arrow heads meeting at R have been slightly displaced. We shall occasionally displace other arrow heads under similar circumstances.)

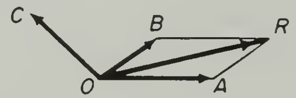
though they do not have the same starting point. Actually, though, we have merely combined $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ by the parallelogram law.* But suppose we were dealing with what are called *free vectors*—vectors having the freedom to move from one location to another, so that $\overrightarrow{OP}$ and $\overrightarrow{QR}$ in Figure 2.2, for example, which have the same magnitude and the same direction, are officially counted not as distinct vectors but as the same free vector. Then we could indeed combine free vectors that were quite far apart by bringing them end-to-end, like $\overrightarrow{OP}$ and $\overrightarrow{PR}$ in Figure 2.2. But since we could also combine them according to the parallelogram law by moving them so that they have a common starting point, like $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ in Figure 2.1, the parallelogram law is the basic one. Note that when we speak of the same direction we mean just that, and not opposite directions—north and south are not the same direction.

*Have you noticed that we have been careless in sometimes speaking of "the vector represented by $\overrightarrow{OP}$," at other times calling it simply "the vector $\overrightarrow{OP}$," and now calling it just "$\overrightarrow{OP}$"? This is deliberate—and standard practice among mathematicians. Using meticulous wording is sometimes too much of an effort once the crucial point has been made.

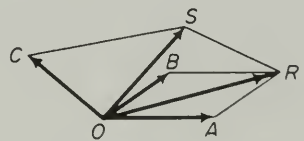
Exercise 2.6 Find the resultant of the three vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$, and $\overrightarrow{OC}$ in the diagram.



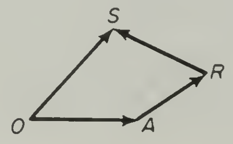
Solution We first form the resultant, $\overrightarrow{OR}$, of $\overrightarrow{OA}$ and $\overrightarrow{OB}$ like this:



and then we form the resultant, $\overrightarrow{OS}$, of $\overrightarrow{OR}$ and $\overrightarrow{OC}$ like this:



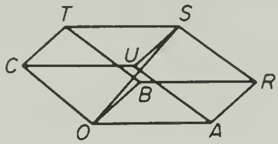
This figure looks complicated. We can simplify it by drawing only half of each parallelogram, and then even omitting the line OR, like this:



From this we see that the resultant $\overrightarrow{OS}$ can be found quickly by thinking of the vectors as free vectors and combining them by placing them end-to-end: $\overrightarrow{AR}$, which has the same magnitude and direction as $\overrightarrow{OB}$, starts where $\overrightarrow{OA}$ ends; and then $\overrightarrow{RS}$, which has the same magnitude and direction as $\overrightarrow{OC}$, starts where $\overrightarrow{AR}$ ends.

Exercise 2.7 Find, by both methods, the resultant of the vectors in Exercise 2.6, but by combining $\overrightarrow{OB}$ and $\overrightarrow{OC}$ first, and then combining their resultant with $\overrightarrow{OA}$. Prove geometrically that the resultant is the same as before.

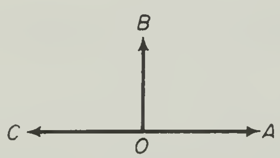
Exercise 2.8



The above diagram looks like a drawing of a box. Show that if we drew only the lines OA, AR, RS, and OS we would have essentially the last figure in Exercise 2.6; that if we drew only the lines OB, BT, TS, and OS we would have a corresponding figure for Exercise 2.7; and that if we drew only OA, AU, US, and OS we would have a figure corresponding to our having first combined $\overrightarrow{OA}$ with $\overrightarrow{OC}$ and then their resultant with $\overrightarrow{OB}$.

Exercise 2.9 In Exercises 2.6, 2.7, and 2.8, is it essential that the three vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$, and $\overrightarrow{OC}$ lie in a plane? Give a rule for finding the resultant of three noncoplanar vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$, and $\overrightarrow{OC}$ that is analogous to the parallelogram law, and that might well be called the parallelepiped law. Prove that their resultant is the same regardless of the order in which one combines them.

Exercise 2.10 Find the resultant of the three vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$, and $\overrightarrow{OC}$ below by combining them in three different orders, given that vectors $\overrightarrow{OA}$ and $\overrightarrow{OC}$ have equal magnitudes and opposite directions. Draw both the end-to-end diagrams and the full parallelogram diagrams for each case.



3. JOURNEYS ARE NOT VECTORS

It is all very well to start with a definition. But it is not very enlightening. Why should scientists and mathematicians be interested in objects that have magnitude and direction and combine according to the parallelogram law? Why did they even think of such objects? Indeed, do such objects exist at all—outside of the imaginations of mathematicians?

There are, of course, many objects that have both magnitude and direction. And there are, unfortunately, many books about vectors that give the reader the impression that such objects obviously and inevitably obey the parallelogram law. It is therefore worthwhile to explain carefully why most such objects do not obey this law, and then, by a process of abstraction, to find objects that do.

Suppose that I live at A and my friend lives 10 miles away at B. I start from A and walk steadily at 4 m.p.h. for $2\frac{1}{2}$ hours. Obviously, I walk 10 miles. But do I reach B?

You may say that this depends on the direction I take. But what reason is there to suppose that I keep to a fixed direction? The chances are overwhelming that I do not—unless I am preceded by a bulldozer or a heavy tank. Most likely I walk in all sorts of directions; and almost certainly, I do not arrive at B. I may even end up at home.

Exercise 3.1 Where are all the possible places at which I can end, under the circumstances?

Now suppose that I start again from A and this time end up at B. I may take four or five hours, or I may go by bus or train and get there quickly. Never mind how I travel or how long I take. Never mind how many times I change my direction, or how tired I get, or how dirty my shoes get, or whether it rained. Ignore all such items, important though they be, and consider the abstraction that results when one concentrates solely on the fact that I start at A and end at B. Let us give this abstraction a name. What shall we call it? Not a "journey." That word reminds us too much of everyday life—of rain, and umbrellas, and vexations, and lovers meeting, and all other such items that we are ignoring here; besides, we want to preserve the word "journey" for just such an everyday concept. For our abstraction we need a neutral, colorless word. Let us call it a *shift*.

Here are routes of four journeys from A to B:

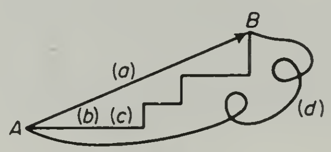


Figure 3.1

All four journeys are different—with the possible but highly improbable exception of (b) and (c).

Exercise 3.2 Why "highly improbable"?

But though the four journeys are not all the same, they yield the same shift. We can represent this shift by the arrow-headed line segment AB. It has both magnitude and direction. Indeed, it seems to have little else. Is it a vector? Let us see.

Consider three places A, B, and C as in Figure 3.2. If I walk in a straight

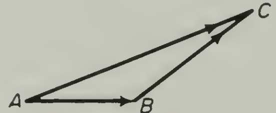


Figure 3.2

line from A to B and then in a straight line from B to C, I make a journey from A to C, but it is not the same as if I walked directly in a straight line from A to C: the scenery is different, and so is the amount of shoe leather consumed, most likely, and we can easily think of several other differences.

Exercise 3.3 Why "most likely"?

Thus, though we could say that the walks from A to B and from B to C combine to give a "resultant" journey from A to C, it is not a journey in a straight line from A to C: the walks do not combine in a way reminiscent of the way in which vectors combine; they combine more in the tautological sense that $2 + 1 = 2 + 1$ than $2 + 1 = 3$.

Journeys, then, are not vectors. But when we deal with shifts we ignore such things as the scenery and the amount of shoe leather consumed. A shift from A to B followed by a shift from B to C is indeed equivalent to a shift from A to C. And this reminds us so strongly of the vectorial situation in Figure 2.2 that we are tempted to conclude that shifts are vectors. But there is a crucial difference between the two situations. We cannot combine the above shifts in the reverse order (compare Exercise 2.5). There is no single equivalent to the shift from B to C followed by the shift from A to B. We can combine two shifts only when the second begins where the first ends. Indeed, in Figure 2.1, just as with journeys, we cannot combine a shift from O to P with one from O to Q in either order. Thus shifts are not vectors.

4. DISPLACEMENTS ARE VECTORS

Now that we have discovered why shifts are not vectors, we can easily see what further abstraction to make to obtain entities that are. From the already abstract idea of a shift, we remove the actual starting point and end point and retain only the relation between them: that B lies such and such a distance from A and in such and such a direction.* Shifts were things we invented in order to bring out certain distinctions. But this new abstraction is an accepted mathematical concept with a technical name: it is called a *displacement*. And it is a vector, as we shall now show.

In Figure 4.1, the arrow-headed line segments AB and LM are parallel and

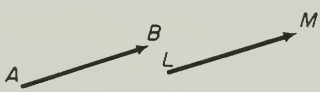


Figure 4.1

of equal length. Any journey from A to B is bound to be different from a journey from L to M. Also, the shift from A to B is different from that from L to M because the starting points are different, as are the end points. But the two shifts, and thus also the various journeys, yield the same displacement: if, for example, B is 5 miles north-northeast of A, so too is M 5 miles north-northeast of L, and the displacement is one of 5 miles in the direction north-northeast.

Exercise 4.1 Starting from a point A, a man bicycles 10 miles due east to point B, stops for lunch, sells his bicycle, and then walks 10 miles due north to point C. Another man starts from B, walks 4 miles due north and 12 miles due east and then, feeling tired, and having brought along

*We retain, too, the recollection that we are still linked, however tenuously, with journeying, for we want to retain the idea that a movement has occurred, even though we do not care at all *how* or under what circumstances it occurred.

a surplus of travellers' checks, buys a car and drives 6 miles due north and 2 miles due west, ending at point D in the pouring rain. What displacement does each man undergo? [*Ans.* $10\sqrt{2}$ miles to the northeast.]

Now look at Figure 2.1. The shift from O to P followed by the shift from P to R is equivalent to the shift from O to R. The shift from P to R gives a displacement $\overrightarrow{PR}$ that is the same as the displacement $\overrightarrow{OQ}$. Therefore the displacement $\overrightarrow{OP}$ followed by the displacement $\overrightarrow{OQ}$ is equivalent to the displacement $\overrightarrow{OR}$.

Exercise 4.2 Prove, similarly, that the displacement $\overrightarrow{OQ}$ followed by the displacement $\overrightarrow{OP}$ is also equivalent to the displacement $\overrightarrow{OR}$.

Thus, displacements have magnitude and direction and combine according to the parallelogram law. According to our definition, they are therefore vectors. Since displacements such as $\overrightarrow{AB}$ and $\overrightarrow{LM}$ in Figure 4.1 are counted as identical, displacements are free vectors, and thus are somewhat special. In general, vectors such as $\overrightarrow{AB}$ and $\overrightarrow{LM}$ are not counted as identical.

5. WHY VECTORS ARE IMPORTANT

From the idea of a journey we have at last come, by a process of successive abstraction, to a specimen of a vector. The question now is whether we have come to anything worthwhile. At first sight it would seem that we have come to so pale a ghost of a journey that it could have little mathematical significance. But we must not underestimate the potency of the mathematical process of abstraction. Vectors happen to be extremely important in science and mathematics. A surprising variety of things happen to have both magnitude and direction and to combine according to the parallelogram law; and many of them are not at all reminiscent of journeys.

This should not surprise us. The process of abstraction is a powerful one. It is, indeed, a basic tool of the mathematician. Take whole numbers, for instance. Like vectors, they are abstractions. We could say that whole numbers are what is left of the idea of *apples* when we ignore not only the apple trees, the wind and the rain, the profits of cider makers, and other such items that would appear in an encyclopedia article, *but also ignore even the apples themselves*, and concentrate solely on how many there are. After we have extracted from the idea of apples the idea of whole numbers, we find that whole numbers apply to all sorts of situations that have nothing to do with apples. Much the same is true of vectors. They are more complicated than whole numbers—so are fractions, for example—but they happen to embody an important type of mathematical behavior that is widely encountered in the world around us.

To give a single example here: forces behave like vectors. This is not something obvious. A force has both magnitude and direction, of course. But this does not mean that forces necessarily combine according to the parallelogram law. That they do combine in this way is inferred from experiment.

It is worthwhile to explain what is meant when we say that forces combine according to the parallelogram law. Forces are not something visible, though their effects may be visible. They are certainly not arrow-headed line segments, though after one has worked with them mathematically for a while, one almost

comes to think they are. A force can be represented by an arrow-headed line segment $\overrightarrow{OP}$ that starts at the point of application O of the force, points in the direction of the force, and has a length proportional to the magnitude of the force—for example, a length of x inches might represent a magnitude of x pounds. When a force is represented in this way, we usually avoid wordiness by talking of "the force $\overrightarrow{OP}$." But let us be more meticulous in our wording just here. To verify experimentally that forces combine according to the parallelogram law, we can make the following experiment. We arrange stationary weights and strings, and pulleys A and B, as shown, the weight W being the

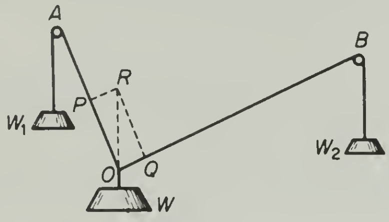


Figure 5.1

sum of the weights W_1 and W_2. Then along OA we mark off a length OP of W_1 inches, where W_1 is the number of pounds in the weight on the left and, thus, a measure of the force with which the string attached to it pulls on the point O where the three pieces of string meet. Similarly, we mark off on OB a length OQ of W_2 inches. We then bring a vertical piece of paper up to the point O, and on it complete the parallelogram defined by OP and OQ. We find that the diagonal OR is vertical and that its length in inches is W, the number of pounds in the weight in the middle. We conclude that the resultant of the forces W_1 and W_2 in the strings would just balance the weight W. Since the forces W_1 and W_2 also just balance the weight W, we say that the resultant is equivalent to the two forces. We then do the experiment over again, with different weights, and reach a similar conclusion. After that, we do it yet again; and we keep at it till our lack of patience overcomes our skepticism, upon which we say that we have proved experimentally that forces combine according to the parallelogram law. And we bolster our assertion by pointing to other experiments, of the same and different types, that indicate the same thing.

We all know that it is much easier to get through a revolving door by pushing near the outer edge than by pushing near the central axis. The effect of a force depends on its location. Home runs are scarce when the bat fails to make contact with the ball. Thus forces do not behave like free vectors. Unlike displacements, vectors representing forces such as $\overrightarrow{AB}$ and $\overrightarrow{LM}$ in Figure 4.1, though they have the same magnitude and the same direction, are not counted as equivalent. Such vectors are called *bound* vectors.

Perhaps it worries us a little that there are different kinds of vectors. Yet we have all, in our time, survived similar complications. Take numbers, for example. There are whole numbers and there are fractions. Perhaps you feel that there is not much difference between the two. Yet if we listed the properties of whole numbers and the properties of fractions we would find considerable differences. For instance, if we divide fractions by fractions the results are always fractions, but this statement does not remain true if we replace the word "fractions" by "whole numbers." Worse, every whole number has a

next higher one, but no fraction has a next higher fraction, for between any two fractions we can always slip infinitely many others. Even so, when trying to define *number* we might be inclined to insist that, given any two different numbers, one of them will always be the smaller and the other the larger. Yet when we become more sophisticated and expand our horizons to include complex numbers like $2 + 3\sqrt{-1}$, we have to give up even this property of being greater or smaller, which at first seemed an absolutely essential part of the idea of number. With vectors too, not only are there various types, but we shall learn that not every one of their attributes that seems at this stage to be essential is in fact so. One of the things that gives mathematics its power is the shedding of attributes that turn out not to be essential, for this, after all, is just the process of abstraction.

Exercise 5.1 Find the resultants of the following displacements:

(a) 3 ft. due east and 3 ft. due north. [*Ans.* $3\sqrt{2}$ ft. to the northeast.]

(b) 5 ft. due north and 5 ft. due east.

(c) 9 cm. to the right and $9\sqrt{3}$ cm. vertically upwards. [*Ans.* 18 cm. in an upward direction making 60° with the horizontal towards the right.]

(d) 9 cm. to the left and $9\sqrt{3}$ cm. vertically downward.

(e) the resultants in parts (c) and (d).

(f) x units positively along the x-axis and y units positively along the y-axis. [*Ans.* $\sqrt{x^2 + y^2}$ units in the direction making an angle $\tan^{-1} y/x$ with the positive x-axis.]

Exercise 5.2 Like Exercise 5.1 for the following:

(a) 8 km. to the left and 3 km. to the left.

(b) 5 fathoms vertically downward and 2 fathoms vertically upward.

(c) α units to the right and β units to the left. [There are three different cases. What are they? Show how they can be summed up in one statement.]

(d) h miles 60° north of east and h miles 60° south of east.

Exercise 5.3 What single force is equivalent to the following three horizontal forces acting on a particle at a point O? (1) magnitude 1 lb. pulling to the north; (2) magnitude 1 lb. pulling to the east; (3) magnitude $\sqrt{2}$ lb. pulling to the northwest. [*Ans.* 2 lbs. acting at point O and pulling to the north.]

Exercise 5.4 What force combined with a force at a point 0 of 1 lb. pulling to the east will yield a resultant force of 2 lbs. pulling in a direction 60° north of east?

Exercise 5.5 Vector $\overrightarrow{OP}$ has magnitude $2a$ and points to the right in a direction 30° above the horizontal. What vector combined with it will yield a vertical resultant, $\overrightarrow{OR}$, of magnitude $2\sqrt{3}\,a$?

Exercise 5.6 Find two forces at a point O, one vertical and one horizontal, that have a resultant of magnitude h, making 45° with the horizontal force. [*Ans.* The forces have magnitude $h/\sqrt{2}$.]

Exercise 5.7 Find two forces at a point O, one vertical and one horizontal, that have a resultant of magnitude h that makes an angle of 30° with the horizontal force.

Exercise 5.8 Find two displacements, one parallel to the x-axis and the other to the y-axis, that yield a resultant displacement of magnitude h ft. making a positive acute angle α with the positive x-direction.

Exercise 5.9 What is the resultant of n vectors, each starting at the point O, each having magnitude h, and each pointing to the pole star? [We could have shortened this by asking for the resultant of n equal vectors. But we have not yet defined "equal" vectors—even though we have already spoken of the equality of free vectors! You may find it instructive to try to do so here; but be warned that it is not as easy as it seems, and that there is something lacking in the wording of the question.]

Exercise 5.10 A particle is acted on by two forces, one of them to the west and of magnitude 1 dyne, and the other in the direction 60° north of east and of magnitude 2 dynes. What third force acting on the particle would keep it in equilibrium (i. e., what third force would make the resultant of all three forces have zero magnitude)? [*Ans.* Magnitude $\sqrt{3}$ dynes pointing due south.]

6. THE SINGULAR INCIDENT OF THE VECTORIAL TRIBE

It is rumored that there was once a tribe of Indians who believed that arrows are vectors. To shoot a deer due northeast, they did not aim an arrow in the northeasterly direction; they sent two arrows simultaneously, one due north and the other due east, relying on the powerful resultant of the two arrows to kill the deer.

Skeptical scientists have doubted the truth of this rumor, pointing out that not the slightest trace of the tribe has ever been found. But the complete disappearance of the tribe through starvation is precisely what one would expect under the circumstances; and since the theory that the tribe existed confirms two such diverse things as the NONVECTORIAL BEHAVIOR OF ARROWS and the DARWINIAN PRINCIPLE OF NATURAL SELECTION, it is surely not a theory to be dismissed lightly.

Exercise 6.1 Arrow-headed line segments have magnitude and direction and are used to represent vectors. Why are they nevertheless not vectors?

Exercise 6.2 Given the three vectors represented by $\overrightarrow{OP}$, $\overrightarrow{OQ}$, and $\overrightarrow{OR}$ in Figure 2.1, form three new entities having the same respective directions, but having magnitudes equal to five times the magnitudes of the respective vectors. Prove geometrically that these new entities are so related that the third is a diagonal of the parallelogram having the other two as adjacent sides.

Exercise 6.3 If in Exercise 6.2 the new entities had the same respective directions as the vectors represented by $\overrightarrow{OP}$, $\overrightarrow{OQ}$, and $\overrightarrow{OR}$, but had magnitudes that were one unit greater than the magnitudes of the corresponding vectors, show that the new entities would not be such that the third was a diagonal of the parallelogram having the other two as adjacent sides.

Exercise 6.4 Suppose we represented vectors by arrow-headed line segments that had the same starting points and directions as the vectors, but had lengths proportional to the squares of the magnitudes of the vectors, so that, for example, if a force of 1 lb. were represented by a segment of length 1 inch, then a force of 2 lbs. would be represented by one of 4 inches. Show that, in general, these representations of vectors would not obey the parallelogram law. Note that the statement of the parallelogram law in Section 2 therefore needs amending, and amend it accordingly. [If you think carefully, you will realize that this is a topsy-turvy question since, in proving the required result, you will assume that the vectors, when "properly" represented, obey the parallelogram law; and thus, in a sense, you will assume the very amendment you are seeking. But since you have probably been assuming the amendment all this while, you will be able to think your way through. The purpose of this exercise is to draw your attention to this rarely mentioned, usually assumed amendment.]

7. SOME AWKWARD QUESTIONS

When are two vectors equal? The answer depends on what we choose to mean by the word "equal"—we are the masters, not the word. But we do not want to use the word in an outrageous sense: for example, we would not want to say that two vectors are equal if they are mentioned in the same sentence.

Choosing a meaning for the word "equal" here is not as easy as one might imagine. For example, we could reasonably say that two vectors having the same magnitudes, identical directions, and a common starting point are equal vectors. And if one of the vectors were somehow pink and the other green, we would probably be inclined to ignore the colors and say that the vectors were still equal. But what if one of the vectors represented a force and the other a displacement? There would then be two difficulties.

The first difficulty is that the vector representing a displacement would be a free vector, but the one representing the force would not. If, in Figure 4.1, we counted free vectors represented by $\overrightarrow{AB}$ and $\overrightarrow{LM}$ as equal, we might find ourselves implying that forces represented by $\overrightarrow{AB}$ and $\overrightarrow{LM}$ were also equal, though actually they have different effects. [Even so, it is extremely convenient to say such things as "a force acts at A and an equal force acts at L." We shall not do so in this book. But one can get by with saying such things once one has explained what is awkward about them, just as, in trigonometry one gets by with writing $\sin^2\theta$ after one has explained that this does not stand for $\sin(\sin\theta)$ but $(\sin\theta)^2$.]

As for the second difficulty about the idea of the equality of vectors, it takes us back to the definition of a vector. For if, in Figure 2.1, $\overrightarrow{OP}$ represents a force and $\overrightarrow{OQ}$ a displacement, the two vectors will not combine by the parallelogram law at all. We know this from experiments with forces. But we can appreciate the awkwardness of the situation by merely asking ourselves what the resultant would be if they did combine in this way. A "disforcement"?* [Compare Exercise 5.9.]

*Actually, of course, lack of a name proves no more than that if the resultant exists, it has not hitherto been deemed important enough to warrant a name.

If two vectors are to be called equal, it seems reasonable to require that they be able to combine with each other. The situation is not the same as it is with numbers. Although 3 apples and 3 colors are different things, we can say that the numbers 3 are equal in the sense that, if we assign a pebble to each of the apples, these pebbles will exactly suffice for doing the same with the colors. And in this sense we can indeed combine 3 apples and 3 colors—not to yield 6 apples, or 6 colors, or 6 colored apples [it would surely be only 3 colored apples], but 6 *items*. There does not seem to be a corresponding sense in which we could reasonably combine a vector representing a force with one representing a displacement, quite apart from the question of bound versus free vectors: there does not seem to be a vectorial analogue of the numerical concept of a countable item.*

Though $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ do not combine according to the parallelogram law if, for example, $\overrightarrow{OP}$ represents a force and $\overrightarrow{OQ}$ a displacement, they nevertheless represent vectors. Evidently our definition of a vector needs even further amendment. We might seek to avoid trouble by retreating to the definition of a vector as "an entity having both magnitude and direction," without mentioning the parallelogram law. But once we start retreating, where do we stop? Why not be content to define a vector as "an entity having direction," or as "an entity having magnitude," or, with Olympian simplicity, as just "an entity"? Alternatively, we could make the important distinction between the abstract mathematical concept of a vector and entities, such as forces, that behave like these abstract vectors and are called *vector quantities*. This helps, but it does not solve the present problem so much as sweep it under the rug. We might amend our definition of a vector by saying that vectors combine according to the parallelogram law only with vectors of the same kind: forces with forces, displacements with displacements, accelerations (which are vectors) with accelerations, and so on. But even that is tricky since, for example, in dynamics we learn that force equals mass times acceleration. So we would have to allow for the fact that though a force does not combine with an acceleration, it does combine with a vector of the type mass-times-acceleration in dynamics.

We shall return to this matter. (See Section 8 of Chapter 2.) But enough of such questions here. If we continue to fuss with the definition we shall never get started. Even if we succeeded in patching up the definition to meet this particular emegency, other emergencies would arise later. The best thing to do is to keep an open mind and learn to live with a flexible situation, and even to relish it as something akin to the true habitat of the best research.

*Even with numbers there are complications. For example, 3 ft. and 3 inches can be said to yield 6 items; yet in another sense they yield 39 inches, $3\frac{1}{4}$ ft., and so on—and each of these can also be regarded as a number of items, though the $3\frac{1}{4}$ involves a further subtlety. Consider also 3 ft. and 3 lbs., and then 2.38477 ft. and 2.38477 lbs.

Galileo uses a thought experiment in discussing projectile motion, a typical device of the scientist to this day. Galileo's book was originally published in 1632.

11 Galileo's Discussion of Projectile Motion

Gerald Holton and Duane H. D. Roller

An excerpt from their book *Foundations of Modern Physical Science,* 1958.

3.1 Galileo's discussion of projectile motion. To this point we have been solely concerned with the motion of objects as characterized by their speed; we have not given much consideration to the *direction* of motion, or to changes in direction of motion. Turning now to the more general problem of projectile motion, we leave the relatively simple case of bodies moving in a *straight line* only and expand our methods to deal with projectiles moving along curved paths. Our understanding of this field will hinge largely on a far-reaching idea: the observed motion of a projectile may be thought of as the result of two *separate* motions, combined and occurring *simultaneously;* one component of motion is in a horizontal direction and without acceleration, whereas the other is in a vertical direction and has a constant acceleration downward in accordance with the laws of free fall. Furthermore, these two components do not interfere with each other; each component may be studied as if the other were not present. Thus the whole motion of the projectile at every moment is simply the result of the two individual actions.

This principle of the independency of the horizontal and vertical components of projectile motion was set forth by Galileo in his *Dialogue on the great world systems* (1632). Although in this work he was principally concerned with astronomy, Galileo already knew that terrestrial mechanics offered the clue to a better understanding of planetary motions. Like the *Two new sciences,* this earlier work is cast in the form of a discussion among the same three characters, and also uses the Socratic method of the Platonic dialogues. Indeed, the portion of interest to us here begins with Salviati reiterating one of Socrates' most famous phrases, as he tells the Aristotelian Simplicio that he, Simplicio, knows far more about mechanics than he is aware:*

Salviati: . . . Yet I am so good a midwife of minds that I will make you confess the same whether you will or no. But Sagredus stands very quiet, and yet, if I mistake not, I saw him make some move as if to speak.

Sagredo: I had intended to speak a fleeting something; but my curiosity

*These extracts from Galileo's *Dialogue on the great world systems,* as well as those appearing in later chapters, are taken from the translation of T. Salusbury, edited and corrected by Giorgio de Santillana (University of Chicago Press, 1953).

aroused by your promising that you would force Simplicius to uncover the knowledge which he conceals from us has made me depose all other thoughts. Therefore I pray you to make good your vaunt.

Salviati: Provided that Simplicius consents to reply to what I shall ask him, I will not fail to do it.

Simplicio: I will answer what I know, assured that I shall not be much put to it, for, of those things which I hold to be false, I think nothing can be known, since Science concerns truths, not falsehoods.

Salviati: I do not desire that you should say that you know anything, save that which you most assuredly know. Therefore, tell me; if you had here a flat surface as polished as a mirror and of a substance as hard as steel that was not horizontal but somewhat inclining, and you put upon it a perfectly spherical ball, say, of bronze, what do you think it would do when released? Do you not believe (as for my part I do) that it would lie still?

Simplicio: If the surface were inclining?

Salviati: Yes, as I have already stated.

Simplicio: I cannot conceive how it should lie still. I am confident that it would move towards the declivity with much propenseness.

Salviati: Take good heed what you say, Simplicius, for I am confident that it would lie still in whatever place you should lay it.

Simplicio: So long as you make use of such suppositions, Salviatus, I shall cease to wonder if you conclude most absurd conclusions.

Salviati: Are you assured, then, that it would freely move towards the declivity?

Simplicio: Who doubts it?

Salviati: And this you verily believe, not because I told you so (for I endeavored to persuade you to think the contrary), but of yourself, and upon your natural judgment?

Simplicio: Now I see your game; you did not say this really believing it, but to try me, and to wrest words out of my mouth with which to condemn me.

Salviati: You are right. And how long and with what velocity would that ball move? But take notice that I gave as the example a ball exactly round, and a plane exquisitely polished, so that all external and accidental impediments might be taken away. Also I would have you remove all obstructions caused by the air's resistance and any other causal obstacles, if any other there can be.

Simplicio: I understand your meaning very well and answer that the ball would continue to move *in infinitum* if the inclination of the plane should last so long, accelerating continually. Such is the nature of ponderous bodies that they acquire strength in going, and, the greater the declivity, the greater the velocity will be.

Simplicio is next led to express his belief that if he observed the ball rolling *up* the inclined plane he would know that it had been pushed or thrown, since it is moving contrary to its natural tendencies. Then Salviati turns to the intermediate case:

Salviati: It seems, then, that hitherto you have well explained to me the accidents of a body on two different planes. Now tell me, what would befall the same body upon a surface that had neither acclivity nor declivity?

Simplicio: Here you must give me a little time to consider my answer. There

being no declivity, there can be no natural inclination to motion; and there being no acclivity, there can be no resistance to being moved. There would then arise an indifference between propulsion and resistance; therefore, I think it ought naturally stand still. But I had forgot myself; it was not long ago that Sagredus gave me to understand that it would do so.

Salviati: So I think, provided one did lay it down gently; but, if it had an impetus directing it towards any part, what would follow?

Simplicio: That it should move towards that part.

Salviati: But with what kind of motion? Continually accelerated, as in declining planes; or successively retarded, as in those ascending?

Simplicio: I cannot tell how to discover any cause of acceleration or retardation, there being no declivity or acclivity.

Salviati: Well, if there be no cause of retardation, even less should there be any cause of rest. How long therefore would you have the body move?

Simplicio: As long as that surface, neither inclined nor declined, shall last.

Salviati: Therefore if such a space were interminate, the motion upon it would likewise have no termination, that is, would be perpetual.

Simplicio: I think so, if the body is of a durable matter.

Salviati: That has been already supposed when it was said that all external and accidental impediments were removed, and the brittleness of the body in this case is one of those accidental impediments. Tell me now, what do you think is the cause that that same ball moves spontaneously upon the inclining plane, and does not, except with violence, upon the plane sloping upwards?

Simplicio: Because the tendency of heavy bodies is to move towards the center of the Earth and only by violence upwards towards the circumference. [This is the kernel of the Scholastic viewpoint on falling bodies (see Section 2.3). Salviati does not refute it, but turns it to Galileo's purposes.]

Salviati: Therefore a surface which should be neither declining nor ascending ought in all its parts to be equally distant from the center. But is there any such surface in the world?

Simplicio: There is no want of it, such is our terrestrial globe, for example, if it were not rough and mountainous. But you have that of the water, at such time as it is calm and still.

Here is the genesis of one of the fundamental principles of the new mechanics: if all "accidental" interferences with an object's motion are removed, the motion will endure. The "accidents" are eliminated in this thought experiment by: (1) proposing the use of a perfectly round, perfectly hard ball on a perfectly smooth surface, and (2) by imagining the surface to be a globe whose surface is everywhere equidistant from the center of the earth, so that the ball's "natural tendency" to go downward is balanced by the upward thrust of the surface. (We shall return to this latter point in our discussion of isolated systems in Chapter 16.) Note carefully the drastic change from the Scholastic view: instead of asking "What makes the ball move?" Galileo asks "What might change its motion?"

Having turned the conversation to smooth water, Galileo brings in the motion of a stone dropping from the mast of a moving ship. Since the stone is moving horizontally with the ship before it is dropped, it should continue to move horizontally while it falls.

Sagredo: If it be true that the impetus with which the ship moves remains indelibly impressed in the stone after it is let fall from the mast; and if it be further true that this motion brings no impediment or retardment to the motion directly downwards natural to the stone, then there ought to ensue an effect of a very wonderful nature. Suppose a ship stands still, and the time of the falling of a stone from the mast's round top to the deck is two beats of the pulse. Then afterwards have the ship under sail and let the same stone depart from the same place. According to what has been premised, it shall still take up the time of two pulses in its fall, in which time the ship will have gone, say, twenty yards. The true motion of the stone then will be a transverse line [i.e., a curved line in the vertical plane, see Fig. 3.1], considerably longer than the first straight and perpendicular line, the height of the mast, and yet nevertheless the stone will have passed it in the same time. Increase the ship's velocity as much as you will, the falling stone shall describe its transverse lines still longer and longer and yet shall pass them all in those selfsame two pulses. In this same fashion, if a cannon were leveled on the top of a tower, and fired point-blank, that is, horizontally, and whether the charge were small or large with the ball falling sometimes a thousand yards distant, sometimes four thousand, sometimes ten, etc., all these shots shall come to ground in times equal to each other. And every one equal to the time that the ball would take to pass from the mouth of the piece to the ground, if, without other impulse, it falls simply downwards in a perpendicular line. Now it seems a very admirable thing that, in the same short time of its falling perpendicularly down to the ground from the height of, say, a hundred yards. equal balls. fired violently out of the piece, should be able to pass four hundred, a thousand, even ten thousand yards. All the balls in all the shots made horizontally remain in the air an equal time [Fig. 3.2].

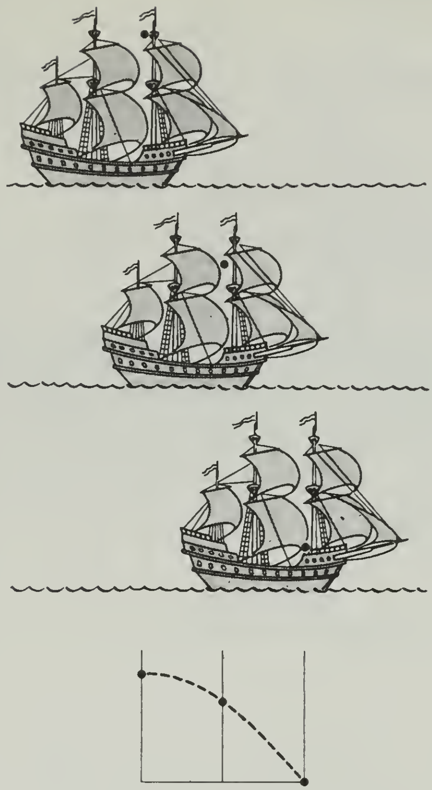


FIG. 3.1. A stone dropped from the mast of a ship in uniform motion. From the shore the trajectory of the stone is seen to be a curved line (parabola).

Salviati: The consideration is very elegant for its novelty and, if the effect be true, very admirable. Of its truth I make no question, and, were it not for the accidental impediment of the air, I verily believe that, if at the time of the ball's going out of the piece another were let fall from the same height directly downwards, they would both come to the ground at the same instant, though one should have traveled ten thousand yards in its range, and another only a hundred, presupposing the surface of the Earth to be level. As for the impedi-

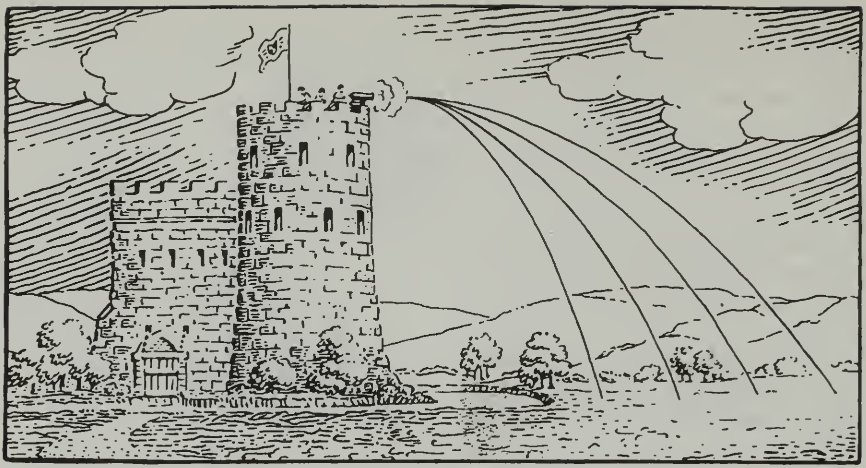


FIG. 3.2. For cannon balls fired horizontally with different initial forward speeds, "all the balls in all the shots made horizontally remain in the air an equal time."

ment which might come from the air, it would consist in retarding the extreme swift motion of the shot.

This chapter from a beginning college physics text is not simple, but the reward of this numerical approach to Newtonian mechanics is a more powerful understanding of how the laws of motion work.

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12 Newton's Laws of Dynamics

Richard P. Feynman, Robert B. Leighton and Matthew Sands

A chapter from their textbook *The Feynman Lectures on Physics, Volume 1*, 1963.

9–1 Momentum and force

The discovery of the laws of dynamics, or the laws of motion, was a dramatic moment in the history of science. Before Newton's time, the motions of things like the planets were a mystery, but after Newton there was complete understanding. Even the slight deviations from Kepler's laws, due to the perturbations of the planets, were computable. The motions of pendulums, oscillators with springs and weights in them, and so on, could all be analyzed completely after Newton's laws were enunciated. So it is with this chapter: before this chapter we could not calculate how a mass on a spring would move; much less could we calculate the perturbations on the planet Uranus due to Jupiter and Saturn. After this chapter we *will* be able to compute not only the motion of the oscillating mass, but also the perturbations on the planet Uranus produced by Jupiter and Saturn!

Galileo made a great advance in the understanding of motion when he discovered the *principle of inertia:* if an object is left alone, is not disturbed, it continues to move with a constant velocity in a straight line if it was originally moving, or it continues to stand still if it was just standing still. Of course this never appears to be the case in nature, for if we slide a block across a table it stops, but that is because it is *not* left to itself—it is rubbing against the table. It required a certain imagination to find the right rule, and that imagination was supplied by Galileo.

Of course, the next thing which is needed is a rule for finding how an object *changes* its speed if something *is* affecting it. That is the contribution of Newton. Newton wrote down three laws: The First Law was a mere restatement of the Galilean principle of inertia just described. The Second Law gave a specific way of determining how the velocity changes under different influences called *forces*. The Third Law describes the forces to some extent, and we shall discuss that at

another time. Here we shall discuss only the Second Law, which asserts that the motion of an object is changed by forces in this way: *the time-rate-of-change of a quantity called momentum is proportional to the force.* We shall state this mathematically shortly, but let us first explain the idea.

Momentum is not the same as *velocity.* A lot of words are used in physics, and they all have precise meanings in physics, although they may not have such precise meanings in everyday language. Momentum is an example, and we must define it precisely. If we exert a certain push with our arms on an object that is light, it moves easily; if we push just as hard on another object that is much heavier in the usual sense, then it moves much less rapidly. Actually, we must change the words from "light" and "heavy" to *less massive* and *more massive*, because there is a difference to be understood between the *weight* of an object and its *inertia.* (How hard it is to get it going is one thing, and how much it weighs is something else.) Weight and inertia are *proportional*, and on the earth's surface are often taken to be numerically equal, which causes a certain confusion to the student. On Mars, weights would be different but the amount of force needed to overcome inertia would be the same.

We use the term *mass* as a quantitative measure of inertia, and we may measure mass, for example, by swinging an object in a circle at a certain speed and measuring how much force we need to keep it in the circle. In this way we find a certain quantity of mass for every object. Now the *momentum* of an object is a product of two parts: its *mass* and its *velocity.* Thus Newton's Second Law may be written mathematically this way:

$$F = \frac{d}{dt}(mv). \tag{9.1}$$

Now there are several points to be considered. In writing down any law such as this, we use many intuitive ideas, implications, and assumptions which are at first combined approximately into our "law." Later we may have to come back and study in greater detail exactly what each term means, but if we try to do this too soon we shall get confused. Thus at the beginning we take several things for granted. First, that the mass of an object is *constant*; it isn't really, but we shall start out with the Newtonian approximation that mass is constant, the same all the time, and that, further, when we put two objects together, their masses *add.* These ideas were of course implied by Newton when he wrote his equation, for otherwise it is meaningless. For example, suppose the mass varied inversely as the velocity; then the momentum would *never change* in any circumstance, so the law means nothing unless you know how the mass changes with velocity. At first we say, *it does not change.*

Then there are some implications concerning force. As a rough approximation we think of force as a kind of push or pull that we make with our muscles, but we can define it more accurately now that we have this law of motion. The most important thing to realize is that this relationship involves not only changes in the *magnitude* of the momentum or of the velocity but also in their *direction.*

If the mass is constant, then Eq. (9.1) can also be written as

$$F = m\frac{dv}{dt} = ma. \tag{9.2}$$

The acceleration a is the rate of change of the velocity, and Newton's Second Law says more than that the effect of a given force varies inversely as the mass; it says also that the *direction* of the change in the velocity and the *direction* of the force are the same. Thus we must understand that a change in a velocity, or an acceleration, has a wider meaning than in common language: The velocity of a moving object can change by its speeding up, slowing down (when it slows down, we say it accelerates with a negative acceleration), or changing its direction of motion. An acceleration at right angles to the velocity was discussed in Chapter 7. There we saw that an object moving in a circle of radius R with a certain speed v along the circle falls away from a straightline path by a distance equal to $\frac{1}{2}(v^2/R)t^2$ if t is very small. Thus the formula for acceleration at right angles to the motion is

$$a = v^2/R, \tag{9.3}$$

and a force at right angles to the velocity will cause an object to move in a curved path whose radius of curvature can be found by dividing the force by the mass to get the acceleration, and then using (9.3).

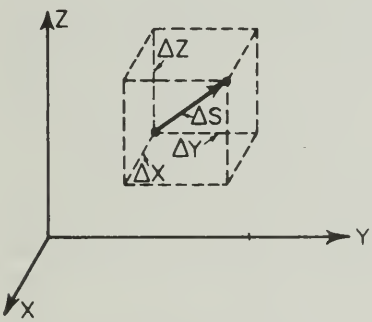

Fig. 9–1. A small displacement of an object.

9–2 Speed and velocity

In order to make our language more precise, we shall make one further definition in our use of the words *speed* and *velocity*. Ordinarily we think of speed and velocity as being the same, and in ordinary language they are the same. But in physics we have taken advantage of the fact that there *are* two words and have chosen to use them to distinguish two ideas. We carefully distinguish velocity, which has both magnitude and direction, from speed, which we choose to mean the magnitude of the velocity, but which does not include the direction. We can formulate this more precisely by describing how the x-, y-, and z-coordinates of an object change with time. Suppose, for example, that at a certain instant an object is moving as shown in Fig. 9–1. In a given small interval of time Δt it

will move a certain distance Δx in the x-direction, Δy in the y-direction, and Δz in the z-direction. The total effect of these three coordinate changes is a displacement Δs along the diagonal of a parallelepiped whose sides are Δx, Δy, and Δz. In terms of the velocity, the displacement Δx is the x-component of the velocity times Δt, and similarly for Δy and Δz:

$$\Delta x = v_x\,\Delta t, \qquad \Delta y = v_y\,\Delta t, \qquad \Delta z = v_z\,\Delta t. \tag{9.4}$$

9–3 Components of velocity, acceleration, and force

In Eq. (9.4) *we have resolved the velocity into components* by telling how fast the object is moving in the x-direction, the y-direction, and the z-direction. The velocity is completely specified, both as to magnitude and direction, if we give the numerical values of its three rectangular components:

$$v_x = dx/dt, \qquad v_y = dy/dt, \qquad v_z = dz/dt. \tag{9.5}$$

On the other hand, the speed of the object is

$$ds/dt = |v| = \sqrt{v_x^2 + v_y^2 + v_z^2}. \tag{9.6}$$

Next, suppose that, because of the action of a force, the velocity changes to some other direction and a different magnitude, as shown in Fig. 9–2. We can analyze this apparently complex situation rather simply if we evaluate the changes in the x-, y-, and z-components of velocity. The change in the component of the velocity in the x-direction in a time Δt is $\Delta v_x = a_x\,\Delta t$, where a_x is what we call the x-component of the acceleration. Similarly, we see that $\Delta v_y = a_y\,\Delta t$ and $\Delta v_z = a_z\,\Delta t$. In these terms, we see that Newton's Second Law, in saying that the force is in the same direction as the acceleration, is really three laws, in the sense that the component of the force in the x-, y-, or z-direction is equal to the mass times

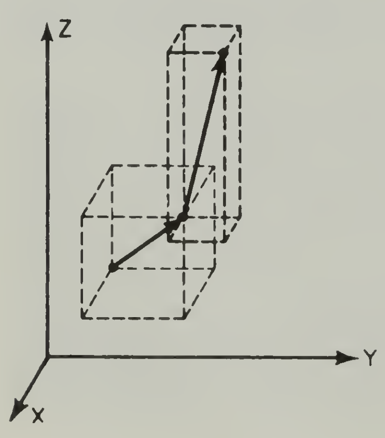

Fig. 9–2. A change in velocity in which both the magnitude and direction change.

the rate of change of the corresponding component of velocity:

$$\begin{aligned} F_x &= m(dv_x/dt) = m(d^2x/dt^2) = ma_x, \\ F_y &= m(dv_y/dt) = m(d^2y/dt^2) = ma_y, \\ F_z &= m(dv_z/dt) = m(d^2z/dt^2) = ma_z. \end{aligned} \tag{9.7}$$

Just as the velocity and acceleration have been resolved into components by projecting a line segment representing the quantity and its direction onto three coordinate axes, so, in the same way, a force in a given direction is represented by certain components in the x-, y-, and z-directions:

$$\begin{aligned} F_x &= F\cos(x, F), \\ F_y &= F\cos(y, F), \\ F_z &= F\cos(z, F). \end{aligned} \tag{9.8}$$

where F is the magnitude of the force and (x, F) represents the angle between the x-axis and the direction of F, etc.

Newton's Second Law is given in complete form in Eq. (9.7). If we know the forces on an object and resolve them into x-, y-, and z-components, then we can find the motion of the object from these equations. Let us consider a simple example. Suppose there are no forces in the y- and z-directions, the only force being in the x-direction, say vertically. Equation (9.7) tells us that there would be changes in the velocity in the vertical direction, but no changes in the horizontal direction. This was demonstrated with a special apparatus in Chapter 7 (see Fig. 7–3). A falling body moves horizontally without any change in horizontal motion, while it moves vertically the same way as it would move if the horizontal motion were zero. In other words, motions in the x-, y-, and z-directions are independent if the *forces* are not connected.

9–4 What is the force?

In order to use Newton's laws, we have to have some formula for the force; these laws say *pay attention to the forces.* If an object is accelerating, some agency is at work; find it. Our program for the future of dynamics must be to *find the laws for the force.* Newton himself went on to give some examples. In the case of gravity he gave a specific formula for the force. In the case of other forces he gave some part of the information in his Third Law, which we will study in the next chapter, having to do with the equality of action and reaction.

Extending our previous example, what are the forces on objects near the earth's surface? Near the earth's surface, the force in the vertical direction due to gravity is proportional to the mass of the object and is nearly independent of height for heights small compared with the earth's radius R: $F = GmM/R^2 = mg$, where $g = GM/R^2$ is called the *acceleration of gravity.* Thus the law of gravity tells us that weight is proportional to mass; the force is in the vertical direction and is the mass times g. Again we find that the motion in the horizontal direction

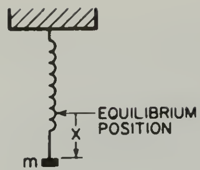

Fig. 9–3. A mass on a spring.

is at constant velocity. The interesting motion is in the vertical direction, and Newton's Second Law tells us

$$mg = m(d^2x/dt^2). \tag{9.9}$$

Cancelling the m's, we find that the acceleration in the x-direction is constant and equal to g. This is of course the well known law of free fall under gravity, which leads to the equations

$$\begin{aligned} v_x &= v_0 + gt, \\ x &= x_0 + v_0t + \tfrac{1}{2}gt^2. \end{aligned} \tag{9.10}$$

As another example, let us suppose that we have been able to build a gadget (Fig. 9–3) which applies a force proportional to the distance and directed oppositely —a spring. If we forget about gravity, which is of course balanced out by the initial stretch of the spring, and talk only about *excess* forces, we see that if we pull the mass down, the spring pulls up, while if we push it up the spring pulls down. This machine has been designed carefully so that the force is greater, the more we pull it up, in exact proportion to the displacement from the balanced condition, and the force upward is similarly proportional to how far we pull down. If we watch the dynamics of this machine, we see a rather beautiful motion—up, down, up, down, . . . The question is, will Newton's equations correctly describe this motion? Let us see whether we can exactly calculate how it moves with this periodic oscillation, by applying Newton's law (9.7). In the present instance, the equation is

$$-kx = m(dv_x/dt). \tag{9.11}$$

Here we have a situation where the velocity in the x-direction changes at a rate proportional to x. Nothing will be gained by retaining numerous constants, so we shall imagine either that the scale of time has changed or that there is an accident in the units, so that we happen to have $k/m = 1$. Thus we shall try to solve the equation

$$dv_x/dt = -x. \tag{9.12}$$

To proceed, we must know what v_x is, but of course we know that the velocity is the rate of change of the position.

9–5 Meaning of the dynamical equations

Now let us try to analyze just what Eq. (9.12) means. Suppose that at a given time t the object has a certain velocity v_x and position x. What is the velocity

and what is the position at a slightly later time $t + \epsilon$? If we can answer this question our problem is solved, for then we can start with the given condition and compute how it changes for the first instant, the next instant, the next instant, and so on, and in this way we gradually evolve the motion. To be specific, let us suppose that at the time $t = 0$ we are given that $x = 1$ and $v_x = 0$. Why does the object move at all? Because there is a *force* on it when it is at any position except $x = 0$. If $x > 0$, that force is upward. Therefore the velocity which is zero starts to change, because of the law of motion. Once it starts to build up some velocity the object starts to move up, and so on. Now at any time t, if ϵ is very small, we may express the position at time $t + \epsilon$ in terms of the position at time t and the velocity at time t to a very good approximation as

$$x(t + \epsilon) = x(t) + \epsilon v_x(t). \tag{9.13}$$

The smaller the ϵ, the more accurate this expression is, but it is still usefully accurate even if ϵ is not vanishingly small. Now what about the velocity? In order to get the velocity later, the velocity at the time $t + \epsilon$, we need to know how the velocity changes, the *acceleration*. And how are we going to find the acceleration? That is where the law of dynamics comes in. The law of dynamics tells us what the acceleration is. It says the acceleration is $-x$.

$$v_x(t + \epsilon) = v_x(t) + \epsilon a_x(t) \tag{9.14}$$

$$= v_x(t) - \epsilon x(t). \tag{9.15}$$

Equation (9.14) is merely kinematics; it says that a velocity changes because of the presence of acceleration. But Eq. (9.15) is *dynamics*, because it relates the acceleration to the force; it says that at this particular time for this particular problem, you can replace the acceleration by $-x(t)$. Therefore, if we know both the x and v at a given time, we know the acceleration, which tells us the new velocity, and we know the new position—this is how the machinery works. The velocity changes a little bit because of the force, and the position changes a little bit because of the velocity.

9–6 Numerical solution of the equations

Now let us really solve the problem. Suppose that we take $\epsilon = 0.100$ sec. After we do all the work if we find that this is not small enough we may have to go back and do it again with $\epsilon = 0.010$ sec. Starting with our initial value $x(0) = 1.00$, what is $x(0.1)$? It is the old position $x(0)$ plus the velocity (which is zero) times 0.10 sec. Thus $x(0.1)$ is still 1.00 because it has not yet started to move. But the new velocity at 0.10 sec will be the old velocity $v(0) = 0$ plus ϵ times the acceleration. The acceleration is $-x(0) = -1.00$. Thus

$$v(0.1) = 0.00 - 0.10 \times 1.00 = -0.10.$$

Now at 0.20 sec

$$\begin{aligned} x(0.2) &= x(0.1) + \epsilon v(0.1) \\ &= 1.00 - 0.10 \times 0.10 = 0.99 \end{aligned}$$

and

$$\begin{aligned} v(0.2) &= v(0.1) + \epsilon a(0.1) \\ &= -0.10 - 0.10 \times 1.00 = -0.20. \end{aligned}$$

And so, on and on and on, we can calculate the rest of the motion, and that is just what we shall do. However, for practical purposes there are some little tricks by which we can increase the accuracy. If we continued this calculation as we have started it, we would find the motion only rather crudely because $\epsilon = 0.100$ sec is rather crude, and we would have to go to a very small interval, say $\epsilon = 0.01$. Then to go through a reasonable total time interval would take a lot of cycles of computation. So we shall organize the work in a way that will increase the precision of our calculations, using the same coarse interval $\epsilon = 0.10$ sec. This can be done if we make a subtle improvement in the technique of the analysis.

Notice that the new position is the old position plus the time interval ϵ times the velocity. But the velocity *when?* The velocity at the beginning of the time interval is one velocity and the velocity at the end of the time interval is another velocity. Our improvement is to use the velocity *halfway between.* If we know the speed now, but the speed is changing, then we are not going to get the right answer by going at the same speed as now. We should use some speed between the "now" speed and the "then" speed at the end of the interval. The same considerations also apply to the velocity: to compute the velocity changes, we should use the acceleration midway between the two times at which the velocity is to be found. Thus the equations that we shall actually use will be something like this: the position later is equal to the position before plus ϵ times the velocity *at the time in the middle of the interval.* Similarly, the velocity at this halfway point is the velocity at a time ϵ before (which is in the middle of the previous interval) plus ϵ times the acceleration at the time t. That is, we use the equations

$$\begin{aligned} x(t + \epsilon) &= x(t) + \epsilon v(t + \epsilon/2), \\ v(t + \epsilon/2) &= v(t - \epsilon/2) + \epsilon a(t), \\ a(t) &= -x(t). \end{aligned} \tag{9.16}$$

There remains only one slight problem: what is $v(\epsilon/2)$? At the start, we are given $v(0)$, not $v(-\epsilon/2)$. To get our calculation started, we shall use a special equation, namely, $v(\epsilon/2) = v(0) + (\epsilon/2)a(0)$.

Now we are ready to carry through our calculation. For convenience, we may arrange the work in the form of a table, with columns for the time, the position, the velocity, and the acceleration, and the in-between lines for the velocity, as shown in Table 9–1. Such a table is, of course, just a convenient way of representing the numerical values obtained from the set of equations (9.16), and in fact the equations themselves need never be written. We just fill in the various spaces in

Table 9–1

Solution of $dv_x/dt = -x$

Interval: $\epsilon = 0.10$ sec

t	x	v_x	a_x
0.0	1.000	0.000	−1.000
		−0.050	
0.1	0.995		−0.995
		−0.150	
0.2	0.980		−0.980
		−0.248	
0.3	0.955		−0.955
		−0.343	
0.4	0.921		−0.921
		−0.435	
0.5	0.877		−0.877
		−0.523	
0.6	0.825		−0.825
		−0.605	
0.7	0.764		−0.764
		−0.682	
0.8	0.696		−0.696
		−0.751	
0.9	0.621		−0.621
		−0.814	
1.0	0.540		−0.540
		−0.868	
1.1	0.453		−0.453
		−0.913	
1.2	0.362		−0.362
		−0.949	
1.3	0.267		−0.267
		−0.976	
1.4	0.169		−0.169
		−0.993	
1.5	0.070		−0.070
		−1.000	
1.6	−0.030		+0.030

the table one by one. This table now gives us a very good idea of the motion: it starts from rest, first picks up a little upward (negative) velocity and it loses some of its distance. The acceleration is then a little bit less but it is still gaining speed. But as it goes on it gains speed more and more slowly, until as it passes $x = 0$ at about $t = 1.50$ sec we can confidently predict that it will keep going, but now it will be on the other side; the position x will become negative, the ac-

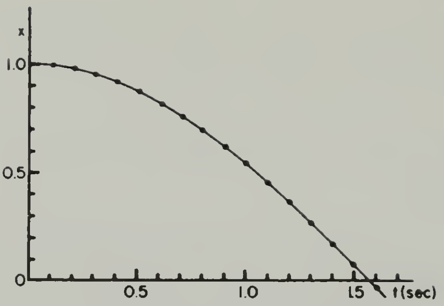

Fig. 9–4. Graph of the motion of a mass on a spring.

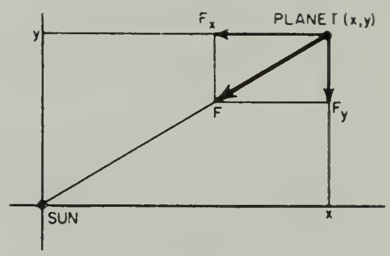

Fig. 9–5. The force of gravity on a planet.

celeration therefore positive. Thus the speed decreases. It is interesting to compare these numbers with the function $x = \cos t$, which is done in Fig. 9–4. The agreement is within the three significant figure accuracy of our calculation! We shall see later that $x = \cos t$ is the exact mathematical solution of our equation of motion, but it is an impressive illustration of the power of numerical analysis that such an easy calculation should give such precise results.

9–7 Planetary motions

The above analysis is very nice for the motion of an oscillating spring, but can we analyze the motion of a planet around the sun? Let us see whether we can arrive at an approximation to an ellipse for the orbit. We shall suppose that the sun is infinitely heavy, in the sense that we shall not include its motion. Suppose a planet starts at a certain place and is moving with a certain velocity; it goes around the sun in some curve, and we shall try to analyze, by Newton's laws of motion and his law of gravitation, what the curve is. How? At a given moment it is at some position in space. If the radial distance from the sun to this position is called r, then we know that there is a force directed inward which, according to the law of gravity, is equal to a constant times the product of the sun's mass and the planet's mass divided by the square of the distance. To analyze this further we must find out what acceleration will be produced by this force. We shall need the *components* of the acceleration along two directions, which we call x and y. Thus if we specify the position of the planet at a given moment by giving x and y (we shall suppose that z is always zero because there is no force in the z-direction and, if there is no initial velocity v_z, there will be nothing to make z other than zero), the force is directed along the line joining the planet to the sun, as shown in Fig. 9–5.

From this figure we see that the horizontal component of the force is related to the complete force in the same manner as the horizontal distance x is to the complete hypotenuse r, because the two triangles are similar. Also, if x is positive,

F_x is negative. That is, $F_x/|F| = -x/r$, or $F_x = -|F|x/r = -GMmx/r^3$. Now we use the dynamical law to find that this force component is equal to the mass of the planet times the rate of change of its velocity in the x-direction. Thus we find the following laws:

$$\begin{aligned} m(dv_x/dt) &= -GMmx/r^3, \\ m(dv_y/dt) &= -GMmy/r^3, \\ r &= \sqrt{x^2 + y^2}. \end{aligned} \tag{9.17}$$

This, then, is the set of equations we must solve. Again, in order to simplify the numerical work, we shall suppose that the unit of time, or the mass of the sun, has been so adjusted (or luck is with us) that $GM \equiv 1$. For our specific example we shall suppose that the initial position of the planet is at $x = 0.500$ and $y = 0.000$, and that the velocity is all in the y-direction at the start, and is of magnitude 1.6300. Now how do we make the calculation? We again make a table with columns for the time, the x-position, the x-velocity v_x, and the x-acceleration a_x; then, separated by a double line, three columns for position, velocity, and acceleration in the y-direction. In order to get the accelerations we are going to need Eq. (9.17); it tells us that the acceleration in the x-direction is $-x/r^3$, and the acceleration in the y-direction is $-y/r^3$, and that r is the square root of $x^2 + y^2$. Thus, given x and y, we must do a little calculating on the side, taking the square root of the sum of the squares to find r and then, to get ready to calculate the two accelerations, it is useful also to evaluate $1/r^3$. This work can be done rather easily by using a table of squares, cubes, and reciprocals: then we need only multiply x by $1/r^3$, which we do on a slide rule.

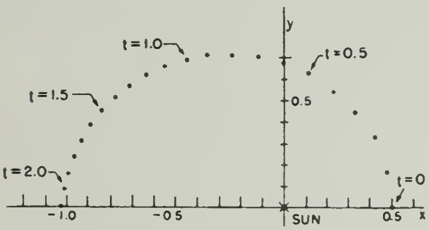

Fig. 9–6. The calculated motion of a planet around the sun.

Our calculation thus proceeds by the following steps, using time intervals $\epsilon = 0.100$: Initial values at $t = 0$:

$$x(0) = 0.500 \qquad y(0) = 0.000$$
$$v_x(0) = 0.000 \qquad v_y(0) = +1.630$$

From these we find:

$$r(0) = 0.500 \qquad 1/r^3(0) = 8.000$$
$$a_x = -4.000 \qquad a_y = 0.000$$

Thus we may calculate the velocities $v_x(0.05)$ and $v_y(0.05)$:

$$\begin{aligned} v_x(0.05) &= 0.000 - 4.000 \times 0.050 = -0.200; \\ v_y(0.05) &= 1.630 + 0.000 \times 0.100 = 1.630. \end{aligned}$$

Now our main calculations begin:

$$\begin{aligned} x(0.1) &= 0.500 - 0.20 \times 0.1 &&= 0.480 \\ y(0.1) &= 0.0 + 1.63 \times 0.1 &&= 0.163 \\ r &= \sqrt{0.480^2 + 0.163^2} &&= 0.507 \\ 1/r^3 &= 7.67 && \\ a_x(0.1) &= 0.480 \times 7.67 &&= -3.68 \\ a_y(0.1) &= -0.163 \times 7.70 &&= -1.256 \\ v_x(0.15) &= -0.200 - 3.68 \times 0.1 &&= -0.568 \\ v_y(0.15) &= 1.630 - 1.26 \times 0.1 &&= 1.505 \\ x(0.2) &= 0.480 - 0.568 \times 0.1 &&= 0.423 \\ y(0.2) &= 0.163 + 1.50 \times 0.1 &&= 0.313 \\ &\text{etc.} && \end{aligned}$$

In this way we obtain the values given in Table 9–2, and in 20 steps or so we have chased the planet halfway around the sun! In Fig. 9–6 are plotted the x- and y-coordinates given in Table 9–2. The dots represent the positions at the succession of times a tenth of a unit apart; we see that at the start the planet moves rapidly and at the end it moves slowly, and so the shape of the curve is determined. Thus we see that we *really do* know how to calculate the motion of planets!

Now let us see how we can calculate the motion of Neptune, Jupiter, Uranus, or any other planet. If we have a great many planets, and let the sun move too, can we do the same thing? Of course we can. We calculate the force on a particular planet, let us say planet number i, which has a position x_i, y_i, z_i ($i = 1$ may represent the sun, $i = 2$ Mercury, $i = 3$ Venus, and so on). We must know the positions of all the planets. The force acting on one is due to all the other bodies which are located, let us say, at positions x_j, y_j, z_j. Therefore the equations are

$$\begin{aligned} m_i \frac{dv_{ix}}{dt} &= \sum_{j=1}^{N} - \frac{Gm_im_j(x_i - x_j)}{r_{ij}^3}, \\ m_i \frac{dv_{iy}}{dt} &= \sum_{j=1}^{N} - \frac{Gm_im_j(y_i - y_j)}{r_{ij}^3}, \\ m_i \frac{dv_{iz}}{dt} &= \sum_{j=1}^{N} - \frac{Gm_im_j(z_i - z_j)}{r_{ij}^3}. \end{aligned} \tag{9.18}$$

Table 9–2

Solution of $dv_x/dt = -x/r^3$, $dv_y/dt = -y/r^3$, $r = \sqrt{x^2 + y^2}$.
Interval: $\epsilon = 0.100$
Orbit $v_y = 1.63 \quad v_x = 0 \quad x = 0.5 \quad y = 0$ at $t = 0$

t	x	v_x	a_x	y	v_y	a_y	r	$1/r^3$
0.0	0.500		−4.00	0.000		0.00	0.500	8.000
		−0.200			1.630			
0.1	0.480		−3.68	0.163		−1.25	0.507	7.675
		−0.568			1.505			
0.2	0.423		−2.91	0.313		−2.15	0.526	6.873
		−0.859			1.290			
0.3	0.337		−1.96	0.442		−2.57	0.556	5.824
		−1.055			1.033			
0.4	0.232		−1.11	0.545		−2.62	0.592	4.81
		−1.166			0.771			
0.5	0.115		−0.453	0.622		−2.45	0.633	3.942
		−1.211			0.526			
0.6	−0.006		+0.020	0.675		−2.20	0.675	3.252
		−1.209			0.306			
0.7	−0.127		+0.344	0.706		−1.91	0.717	2.712
		−1.175			0.115			
0.8	−0.245		+0.562	0.718		−1.64	0.758	2.296
		−1.119			−0.049			
0.9	−0.357		+0.705	0.713		−1.41	0.797	1.975
		−1.048			−0.190			
1.0	−0.462		+0.796	0.694		−1.20	0.834	1.723
		−0.968			−0.310			
1.1	−0.559		+0.858	0.663		−1.02	0.867	1.535
		−0.882			−0.412			
1.2	−0.647		+0.90	0.622		−0.86	0.897	1.385
		−0.792			−0.499			
1.3	−0.726		+0.92	0.572		−0.72	0.924	1.267
		−0.700			−0.570			
1.4	−0.796		+0.93	0.515		−0.60	0.948	1.173
		−0.607			−0.630			
1.5	−0.857		+0.94	0.452		−0.50	0.969	1.099
		−0.513			−0.680			
1.6	−0.908		+0.95	0.384		−0.40	0.986	1.043
		−0.418			−0.720			
1.7	−0.950		+0.95	0.312		−0.31	1.000	1.000
		−0.323			−0.751			
1.8	−0.982		+0.95	0.237		−0.23	1.010	0.970
		−0.228			−0.773			
1.9	−1.005		+0.95	0.160		−0.15	1.018	0.948
		−0.113			−0.778			
2.0	−1.018		+0.96	0.081		−0.08	1.021	0.939
		−0.037			−0.796			
2.1	−1.022		+0.95	0.001		0.00	1.022	0.936
		+0.058			−0.796			
2.2	−1.016		+0.96	−0.079		+0.07	1.019	0.945
					−0.789			
2.3								

Crossed x-axis at 2.101 sec, $\therefore$ period $= 4.20$ sec.

$v_x = 0$ at 2.086 sec.

Cross x at 1.022, $\therefore$ semimajor axis $= \dfrac{1.022 + 0.500}{2} = 0.761.$

$v_y = 0.796.$

Predicted time $\pi(0.761)^{3/2} = \pi(0.663) = 2.082.$

Further, we define r_{ij} as the distance between the two planets i and j; this is equal to

$$r_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}. \tag{9.19}$$

Also, $\sum$ means a sum over all values of j—all other bodies—except, of course, for $j = i$. Thus all we have to do is to make more columns, *lots* more columns. We need nine columns for the motions of Jupiter, nine for the motions of Saturn, and so on. Then when we have all initial positions and velocities we can calculate all the accelerations from Eq. (9.18) by first calculating all the distances, using Eq. (9.19). Hov. long will it take to do it? If you do it at home, it will take a very long time! But in modern times we have machines which do arithmetic very rapidly; a very good computing machine may take 1 microsecond, that is, a millionth of a second, to do an addition. To do a multiplication takes longer, say 10 microseconds. It may be that in one cycle of calculation, depending on the problem, we may have 30 multiplications, or something like that, so one cycle will take 300 microseconds. That means that we can do 3000 cycles of computation per second. In order to get an accuracy, of, say, one part in a billion, we would need 4×10^5 cycles to correspond to one revolution of a planet around the sun. That corresponds to a computation time of 130 seconds or about two minutes. Thus it take only two minutes to follow Jupiter around the sun, with all the perturbations of all the planets correct to one part in a billion, by this method! (It turns out that the error varies about as the square of the interval ϵ. If we make the interval a thousand times smaller, it is a million times more accurate. So, let us make the interval 10,000 times smaller.)

So, as we said, we began this chapter not knowing how to calculate even the motion of a mass on a spring. Now, armed with the tremendous power of Newton's laws, we can not only calculate such simple motions but also, given only a machine to handle the arithmetic, even the tremendously complex motions of the planets, to as high a degree of precision as we wish!

An experimental study of a complex motion, that of a golf club, is outlined. If you do not have a slow-motion movie camera, similar measurements can be made using the stroboscopic picture.

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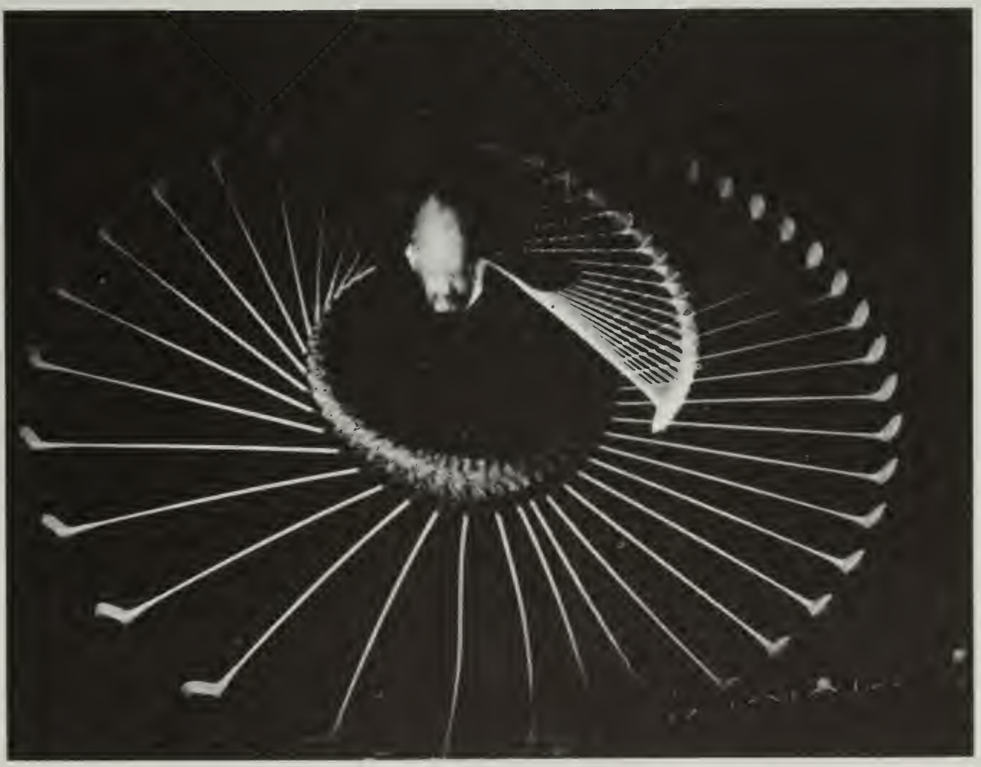
13 The Dynamics of a Golf Club

C. L. Stong

An article from *Scientific American*, 1964.



With the aid of a slow-motion movie camera and a co-operative friend any golf player can easily explore the dynamics of his club head during the split second of the drive that separates the sheep from the goats of golfdom. The procedure, as applied by Louis A. Graham, a consulting engineer in Naples, Fla., analyzes the travel of the club head throughout the swing, including its velocity and acceleration at the critical moment of impact—factors that determine whether a squarely struck ball will merely topple off the tee or go a history-making 445 yards to match the performance of E. C. Bliss in August, 1913.

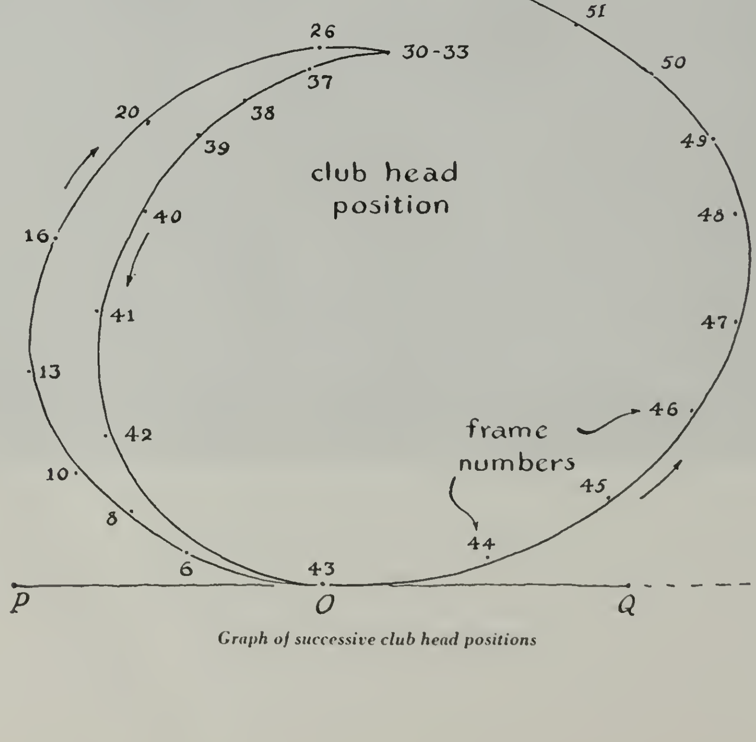
"The procedure is essentially simple," writes Graham, "but the reliability of the results will reflect the care with which certain measurements are made. I pick a sunny day for the experiment and, having arrived at the golf course with my co-operative friend and accessories, tee my ball. Then I place a tee marker precisely four feet in front of the ball and another four feet behind it to make a line that points toward the first green. My friend stations the tripod-mounted camera for a medium close-up shot on a line that intersects the ball at right angles to the tee markers. I address the ball, facing the camera. My friend photographs the complete drive from address to follow-through at the rate of 48 frames per second. The known distance between the tee markers and their position in relation to the club head scales the pictures with respect to distance. The exposure rate—the number of frames per second—of the camera provides the time dimension. (If the exposure rate is not known accurately, it can be calibrated by photographing a phonograph turntable marked with a chalk line and turning at 45 or 78 revolutions per minute.)

"The film is developed and analyzed. One can use either a film-editing device that projects an enlarged image of each frame or a set of enlarged prints of each frame, mounted serially and numbered for identification.

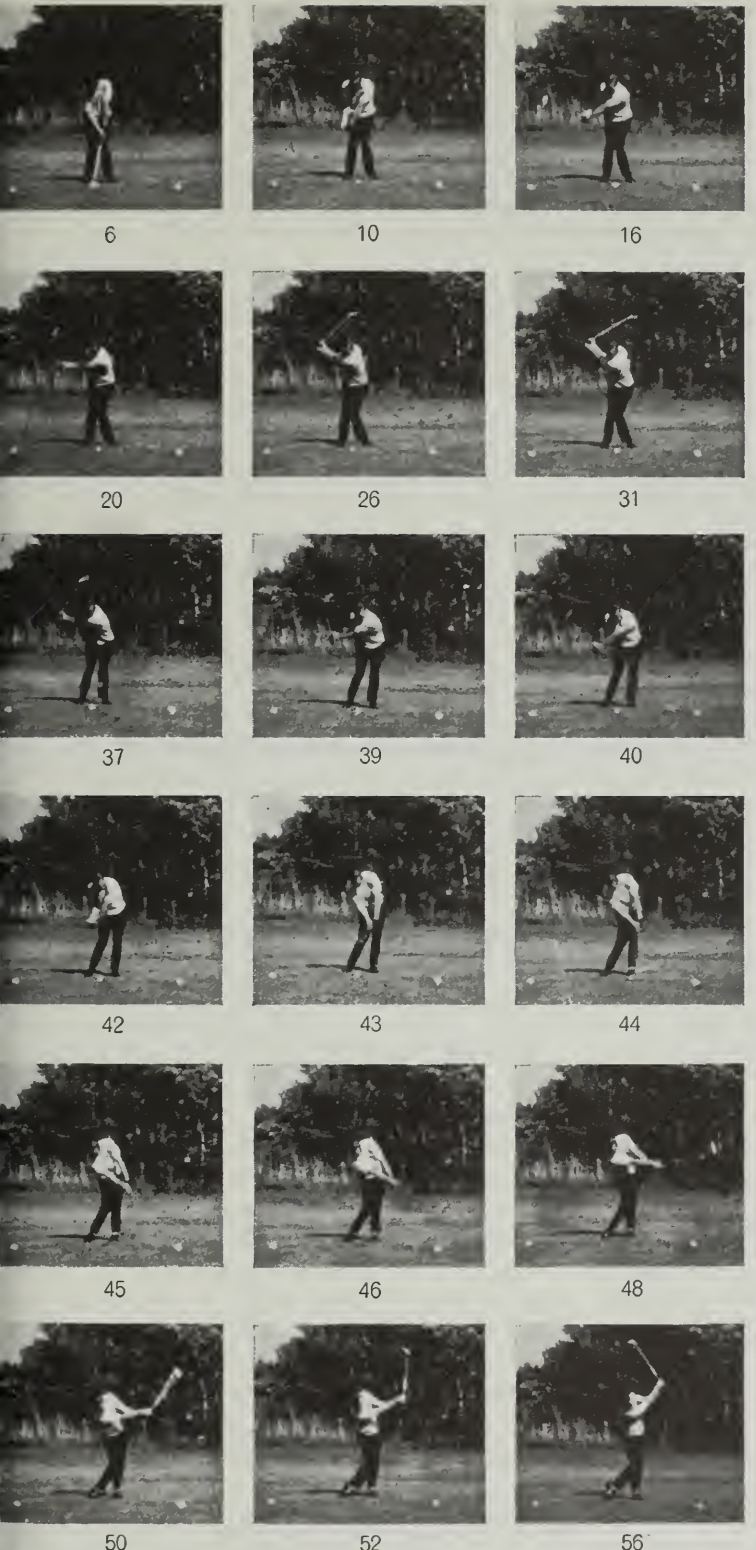
"The next step is to plot the position of the club head during the course of the swing. Since a point in a plane is determined by its distance from two other known points, the position of the club head can be plotted in relation to that of the two tee markers [*see illustration below*]. First, I draw a base line near the bottom of a sheet of graph paper ruled with rectangular co-ordinates and on it locate three equally spaced points: the tee marker *P*, the ball (*O*) and the tee marker *Q*. I usually space these points four inches apart, thus establishing a scale of 12 inches of club head travel per inch of graph paper.

"The location of the club head (*C*) with respect to that of the tee markers can be transferred to the graph by one of three methods. Proportional dividers are handy for transferring the scaled distance from *P* to *C* and from *C* to *Q*. Alternatively, the angles *CPQ* and *CQP* can be measured with a protractor and reconstructed on the graph, point *C* being located at the intersection of lines projected from *P* and *Q*. If no protractor is at hand, the vertical and horizontal distances between *C*, *P* and *Q* can be measured with a square and ruler and similarly transferred to the graph.

"Plot enough points to establish a reasonably smooth track, skipping several frames during slow portions of the swing. The resulting graph is of course not extremely accurate. The plane in which the club head swings, for example, is inclined to the plane of the film. The track plotted from the image therefore differs slightly from the true excursion of the club head, but the error is not large and can be ignored. By the same token, the travel of the club head from



Graph of successive club head positions



Selected frames from slow-motion film of a golf swing

point to point is subsequently measured along straight lines, whereas the club head actually follows a curved path. Error introduced by this source can be minimized by speeding up the camera. My camera, an inexpensive one, is limited to a maximum speed of 48 frames per second, a rate that records the event adequately for the objectives of this experiment.

"The total distance traveled by the club head and its velocity and acceleration are derived from a second set of graphs prepared from the graph of club head position. On a second sheet of graph paper ruled with rectangular coordinates divide the abscissa into a series of uniform increments equal to the total number of frames occupied by the swing and note the corresponding time intervals in seconds as well as the frame numbers. The ordinate will carry two scales: club head travel in feet and club head speed in miles per hour. The scales of the ordinate should provide for a total club head travel of 36 feet and a maximum velocity of about 80 miles per hour. Graphs of convenient proportion result when the length of the ordinate representing 36 feet equals the length representing one second on the abscissa. The maximum velocity of 80 miles per hour need not occupy more than half of the ordinate scale, as shown in the accompanying graph [*upper illustration on page* 94].

"Data for plotting club head travel against time are derived by measuring the graph of club head position. Make a table of three columns, for frame number, time and distance. Beginning with the point on the graph of club head travel that shows the head addressing the ball, scale the distance to the next point and convert to equivalent feet by referring the measurement to the base line that includes *P*, *O* and *Q*. Measure and tabulate the remaining position points in the same way. When the table is complete, add the distance increments progressively, plot distance against time and draw a smooth curve through the points.

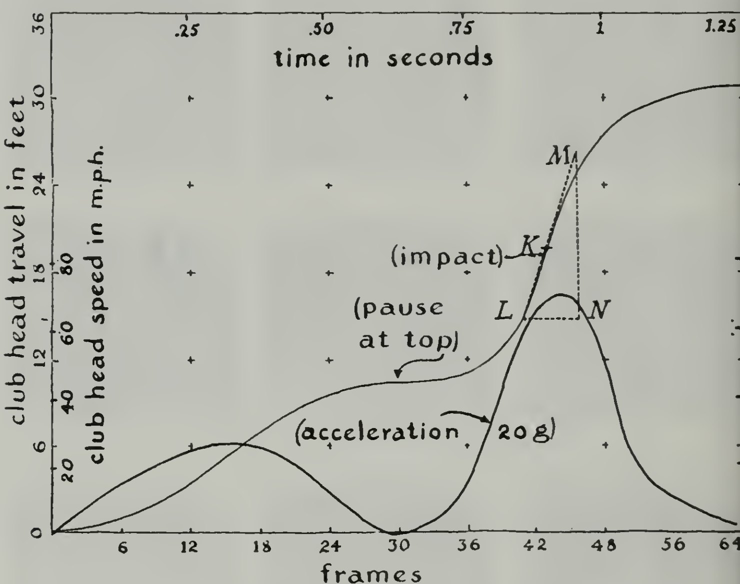
"The speed of the club head at any point is found from this graph by the familiar graphical method of slopes. To find the speed of the club head at about the point of impact (frame No. 43), draw a tangent *LKM* of arbitrary length through *K*. The sides *MN* and *LN* are found by referring to the scale to equal 11.2 feet and .11 second respectively. The speed of the club head at this instant is equal to the ratio 11.2/.11, or 102 feet per second. The result can be expressed in miles per hour by multi-

plying it by the number of seconds per hour and dividing the product by the number of feet per mile: 102 × 3,600/5,280 = 70 miles per hour. Repeat the procedure for each of the frames, tabulate the results, plot speed versus time and draw a smooth curve through the points.

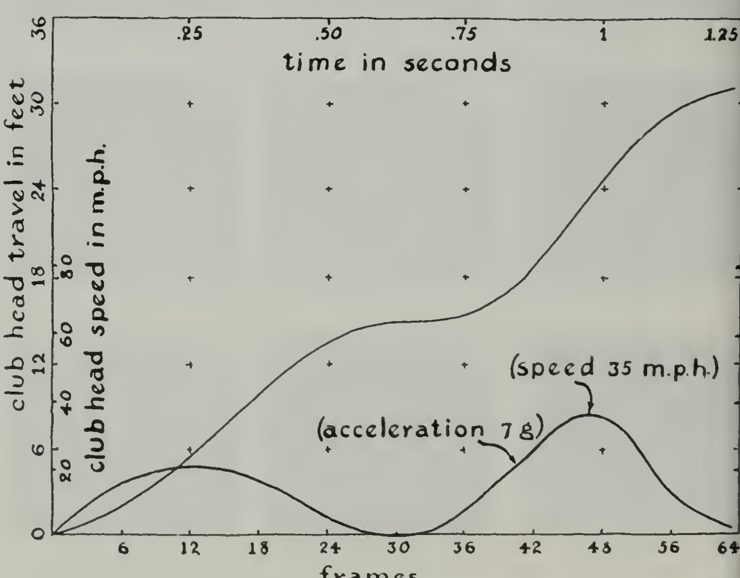
"Club head acceleration can be graphed in the same way or merely computed from the graph of club head speed at frames of particular interest, such as the frame showing the moment of impact. For example, to determine the acceleration of the club head depicted by frame No. 38, draw a tangent to the graph at this point. Then, at some arbitrary point above, say at the point corresponding to a velocity of 56 miles per hour, drop a perpendicular *MN* from the tangent. At another arbitrary point below, say at the point corresponding to a velocity of 12 miles per hour, draw a line *LN* parallel to the abscissa and intersecting both the tangent and *MN*. Inspection of the abscissa discloses that the length *LN* is analogous to a time interval of .1 second. Acceleration is defined as the rate of change of velocity and is equal to the difference between the final velocity and initial velocity divided by the time interval between the two. In this example the velocity difference is 56 miles per hour minus 12 miles per hour, or, expressed in feet per second: (56 − 12) × 5,280/3,600 = 64 feet per second. The acceleration is 64/.1 = 640 feet per second per second. The acceleration of gravity (g) amounts to 32 feet per second per second. The acceleration of the club head at frame No. 38 in terms of g is accordingly 640/32, or 20 g!

"Having performed this rainy-afternoon portion of the procedure, what reward awaits the duffer? For one thing, he can see at a glance why his drives do not match those of a professional golfer. The graphs discussed so far show the performance of golf professional Dick Bull using an iron. His swing from address to follow-through required 1.17 seconds. The club head traveled 31 feet. His backswing occupied .6 second. He paused at the top about .1 second. More interesting than these figures, in my opinion, are those of the club head speed and acceleration Bull achieved: the increase in club head speed during the .1 second before impact from 15 miles per hour to an amazing 70 miles per hour, representing an acceleration of slightly over 20 g. Graphs of Bull's performance with a driver, although different in many respects from those of his irons, show exactly the same figure for speed, 70 miles per hour, and an acceleration of 22 g, a remarkably uniform performance. Similar analysis of the performance of a fairly good amateur using a driver shows precisely half the velocity of Bull's club, 35 miles per hour, and an acceleration at impact of only seven g [*see lower illustration below*].

"Although these methods of analyzing motion are routine in engineering circles, I am not familiar with their prior application to the game of golf. As with many procedures, they are easier to apply than to describe. I find them interesting because they clearly reveal why Bull and other professionals achieve their long drives. Duffers with movie cameras may well begin asking each other, 'How's your v and g?' "



Speed and acceleration graph for a professional's swing



Similar graph for an amateur's performance

Athletic events involve measurements of distance and time, and so bring in the same error considerations that one also meets in the laboratory.

14 Bad Physics in Athletic Measurements

P. Kirkpatrick

An article from *The American Journal of Physics*, 1944.

THE physics teacher has been accustomed to find in athletic activities excellent problems involving velocities, accelerations, projectiles and impacts. He has at the same time overlooked a rich source of illustrations of fictitious precision and bad metrology. When the student is told that the height of a tree should not be expressed as 144.632 ft if the length of its shadow has been measured only to the nearest foot, the student may see the point at once and yet ask, "What difference does it make?" But when shown that common procedures in measuring the achievements of a discus thrower could easily award a world's record to the wrong man, the student agrees that good technic in measurement is something more than an academic ideal. The present discussion[1] has been prepared partly to give the physics teacher something to talk about, but also to start a chain of publicity which may ultimately make athletic administrators better physicists and so make their awards more just.

If physicists were given charge of the measurements of sport, one may feel sure that they would frown upon the practice of announcing the speed of a racing automobile in six or seven digits—see, for example, the *World Almanac* for any year—when neither the length of the course nor the elapsed time is known one-tenth so precisely. They could and would point out such inconsistencies as that observed in some of the events of the 1932 Olympic games when races were electrically and photographically timed to 0.01 sec, but with the starting gun fired from such a position that its report could not reach the ears of the waiting runners until perhaps 0.03 to 0.04 sec after the official start of the race. In this case, electric timing was used only as an unofficial or semi-official supplement to 0.1-sec hand timing; but it is easy to see that a systematic error of a few hundredths of a second will frequently cause stopwatch timers to catch the wrong tenth.

Scientific counsel on the field would immediately advise judges of the high jump and pole vault to measure heights from the point of take-off instead of from an irrelevant point directly below the bar which should be at the same level but sometimes isn't. Physicists would suggest equipping field judges with surveying instruments for determining after each throw, not only how far the weight traveled but also the relative

[1] Some of the material in this article appeared in a paper by the author in *Scientific American*, April 1937, and is incorporated here by permission of the editors.

elevation of the landing point and the throwing circle. Certainly it is meaningless if not deceptive to record weight throws to a small fraction of an inch when surface irregularities may be falsifying by inches the true merit of the performance.

In shot-putting, for example, a measured length will be in error by practically the same amount as the discrepancy between initial and final elevations, since the flight of the shot at its terminus is inclined at about 45° to the horizontal. For the discus the effect is some three times as serious because of the flatter trajectory employed with this missile, while broad jumpers under usual conditions must be prepared to give or take as much as 0.5 ft, according to the luck of the pit. Meanwhile, the achievements in these events go down in the books with the last eighth or even the last sixteenth of an inch recorded.

At the 1932 Olympic Games an effective device was used to grade the broad-jumping pit to the level of the take-off board before each leap, but the practice has not become general. Athletic regulations, indeed, recognize the desirability of proper leveling in nearly all the field events, but in actual usage not enough is done about it. Since sprinters are not credited with records achieved when blown along before the wind, there is no obvious reason why weight hurlers should be permitted to throw things down hill.

The rule books make no specification as to the hardness of the surface upon which weights shall be thrown, but this property has a significant effect upon the measured ranges of the shot and hammer, since it is prescribed that measurement shall be made to the near side of the impression produced by the landing weight. In a soft surface this impression may be enlarged in the backward direction enough to diminish the throw by several times the ostensible precision of the measurement.

A physicist would never check the identity of three or four iron balls as to mass by the aid of grocers' scales or the equivalent and then pretend that there was any significance in the fact that one of them was thrown a quarter of an inch farther than the others. In measuring the length of a javelin throw, no physicist who wanted to be right to $\frac{1}{8}$ in. would be content to establish his perpendicular from the point of fall to the scratchline by a process of guesswork, but this is the way it is always done by field judges, even in the best competition.

Among the numerous errors afflicting measurements in the field sports, there is none which is more systematically committed, or which could be more easily rectified, than that pertaining to the variation of the force of gravity. The range of a projectile dispatched at any particular angle of elevation and with a given initial speed is a simple function of g. Only in case the end of the trajectory is at the same level as its beginning does this function become an inverse proportionality; but in any case the relationship is readily expressed, and no physicist will doubt that a given heave of the shot will yield a longer put in equatorial latitudes than it would in zones where the gravitational force is stronger. Before saying that the 55-ft put achieved by A in Mexico City is a better performance than one of 54 ft, 11 in. which B accomplished in Boston, we should surely inquire about the values of g which the respective athletes were up against, but it is never done. As a matter of record, the value of g in Boston exceeds that in Mexico City by $\frac{1}{4}$ percent, so the shorter put was really the better. To ignore the handicap of a larger value of g is like measuring the throw with a stretched tape. The latter practice would never be countenanced under AAU or Olympic regulations, but the former is standard procedure.

Rendering justice to an athlete who has had to compete against a high value of g involves questions that are not simple. It will be agreed that he is entitled to some compensation and that in comparing two throws made under conditions similar except as to g, the proper procedure would be to compare not the actual ranges achieved, but the ranges which would have been achieved had some "standard" value of g—say 980 cm/sec^2—prevailed in both cases. The calculation of exactly what would have happened is probably impossible to physics. Although it is a simple matter to discuss the behavior of the implement after it leaves the thrower's hand and to state how this behavior depends upon g, the dependence of the initial velocity of projection upon g depends upon the thrower's form and upon characteristics of body mechanics to which but little attention has so far been devoted.

The work done by the thrower bestows upon the projectile both potential and kinetic energy. In a strong gravitational field, the imparted potential energy is large and one must therefore suppose the kinetic energy to be reduced, since the thrower's propelling energy must be distributed to both. We have no proof, however, that the *total* useful work is constant despite variation of g, nor do we know the manner of its inconstancy, if any. The muscular catapult is not a spring, subject to Hooke's law, but a far more complicated system with many unknown characteristics. The maximum external work which one may do in a single energetic shove by arms, legs or both obviously depends partly upon the resisting force encountered. Only a little outside work can be done in putting a ping-pong ball because the maximum possible acceleration, limited by the masses and other characteristics of the bodily mechanism itself, is too slight to call out substantial inertial forces in so small a mass. The resisting force encountered when a massive body is pushed in a direction that has an upward component, as in shot-putting, does of course depend upon g; and until we know from experiment how external work in such an effort varies with resisting force, we shall not be able to treat the interior ballistics of the shot-putter with anything approaching rigor.

Several alternative assumptions may be considered. If we suppose that the *velocity* of delivery, or "muzzle velocity," v, of the missile is unaffected by variations of g, we have only the external effect to deal with. Adopting the approximate range formula $R=v^2/g$ (which neglects the fact that the two ends of the trajectory are at different levels and which assumes the optimum angle of elevation) we find that the increment of range $\mathrm{d}R$ resulting from an increment $\mathrm{d}g$ is simply $-R\mathrm{d}g/g$. On the more plausible assumption that the *total work done on the projectile* is independent of g, this total to include both the potential and kinetic energies imparted, one obtains as a correction formula,

$$\mathrm{d}R=-\left(1+\frac{2h}{R}\right)R\frac{\mathrm{d}g}{g}, \qquad (1)$$

where h is the vertical lift which the projectile gets while in the hand of the thrower. A third assumption, perhaps the most credible of all, would hold constant and independent of g the *total work done upon the projectile and upon a portion of the mass of the thrower's person.* It is not necessary to decide how much of the thrower's mass goes into this latter term; it drops out and we have again Eq. (1), provided only that the work done on the thrower's body can be taken into account by an addition to the mass of the projectile.

These considerations show that a variation of g affects the range in the same sense before and after delivery, an increase in g reducing the delivery velocity and also pulling the projectile down more forcibly after its flight begins. They indicate also that the latter effect is the more important since, in Eq. (1), $1>2h/R$ by a factor of perhaps five in the shot-put and more in the other weight-throwing events.

One concludes that the *least* which should be done to make amends to a competitor striving against a large value of g is to give him credit for the range which his projectile would have attained, for the same initial velocity, at a location where g is "standard." This is not quite justice, but it is a major step in the right direction. The competitor who has been favored by a small value of g should of course have his achievement treated in the same way.

The corrections so calculated will not be negligible magnitudes, as Fig. 1 shows. They are extremely small percentages of the real ranges, but definitely exceed the ostensible probable errors of measurement. It is not customary to state probable errors explicitly in connection with athletic measurements, but when a throw is recorded as 57 ft, $1\frac{5}{32}$ in., one naturally concludes that the last thirty-second inch, if not completely reliable, must have been regarded as having *some* significance.

ROTATION OF THE EARTH

It is customary to take account of the effects of terrestrial rotation when aiming long-range guns, but athletes and administrators of sport have given little or no attention to such effects in relation to their projectiles. As a matter of fact they should, for at low latitudes the range of a discus or shot thrown in an eastward direction

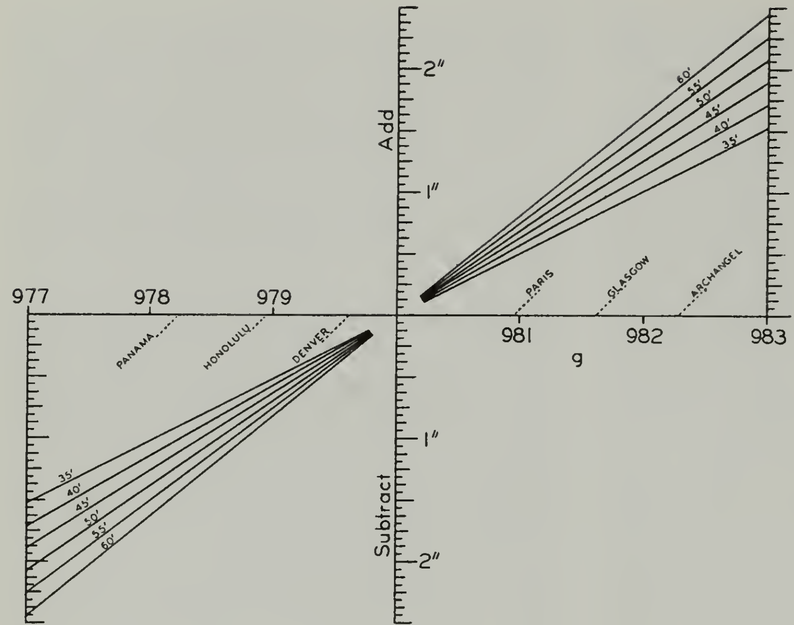


FIG. 1. Graphs for normalizing shot-put ranges to the common value $g=980$ cm/sec². Ranges achieved where $g=980$ cm/sec² are not in need of adjustment, but a range of 50 ft (see inclined line marked 50′) achieved at Glasgow, where $g=981.6$ cm/sec², is entitled to a premium of $1\frac{1}{8}$ in. which should be added before comparing the put with one achieved elsewhere. Distances accomplished where $g<980$ cm/sec² should be subjected to the deductions indicated by graphs in the third quadrant.

exceeds that of a westward throw by more than the ostensible precision of such measurements. The difference between the range of a projectile thrown from the surface of the real earth and the range of one thrown from a nonrotating earth possessing the same local value of g is given by[2]

$$\text{Range}=\frac{V_0^2 \sin 2\alpha}{g}+\frac{4\omega V_0^3}{3g^2} \times \sin \alpha[4 \cos^2 \alpha-1] \cos \lambda \sin \mu, \quad (2)$$

where g is the ordinary acceleration due to weight, V_0 is the initial speed of the projectile, α is the angle of elevation of initial motion (measured upward from the horizontal in the direction of projection), ω(rad/sec) is the angular speed of rotation of the earth, λ is the geographic latitude of the point of departure of the projectile, and μ is the azimuth of the plane of the trajectory, measured clockwise from the north point.

A derivation of this equation (though not the first) is given in reference 2, along with a discussion of its application to real cases. The approximations accepted in the derivation are such as might possibly be criticized where long-range guns are considered, but they introduce no measurable errors into the treatment of athletic projectiles.

The first term of the right-hand member of Eq. (2) is the ordinary elementary range expression, and naturally it expresses almost the whole of the actual range. The second term is a small correction which is of positive sign for eastbound projectiles ($0<\mu<180°$) and negative for westbound. The correction term, being proportional to V_0^3, increases with V_0 at a greater rate than does the range as a whole. Hence the *percentage* increase or decrease of range, because of earth rotation, varies in proportion to V_0 or to the square root of the range itself. Evidently this effect is a maximum at the equator and zero at the poles. Inspection of the role of α shows that the correction term is a maximum for a 30° angle of elevation and that it vanishes when the angle of elevation is 60°.

By the appropriate numerical substitutions in Eq. (2), one may show that a well-thrown discus in tropic latitudes will go an inch farther eastward than westward. This is many times the apparent precision of measurement for this event, and records have changed hands on slimmer margins. Significant effects of the same kind, though of lesser magnitude, appear in the cases

[2] P. Kirkpatrick, Am. J. Phys. 11, 303 (1943).

of the javelin, hammer, shot and even the broad jump, where the east-west differential exceeds the commonly recorded sixteenth of an inch.

Figures 1 and 2 are types of correction charts that might be used to normalize the performances of weight throwers to a uniform value of g and a common direction of projection. Figure 1 has been prepared with the shot-put in mind, but is not restricted to implements of any particular mass. The inclined straight lines of this figure are graphs of $-\mathrm{d}R$ *versus* $\mathrm{d}g$ from Eq. (1). Values of the parameter R are indicated on the graphs. The uniform value 100 cm has been adopted for h, an arbitrary procedure but a harmless one in view of the insensitivity of $\mathrm{d}R$ to h.

Figure 2, particularly applicable to the hammer throw, furnishes means for equalizing the effect of earth spin upon athletes competing with the same implement but directing their throws variously as may be necessitated by the lay-out of their respective fields. An angle of elevation of 45° has been assumed in the construction of these curves, a somewhat restrictive procedure which finds justification in the fact that no hammer thrown at an angle significantly different from 45° is likely to achieve a range worth correcting. These curves are plotted from Eq. (2); their application to particular cases is described in the figure legend.

Upon noticing that some of these corrections are quite small fractions of an inch, the reader may ask whether the trouble is worth while. This is a question that is in great need of clarification and one that may not be answered with positiveness until the concept of the probable error of a measurement shall have become established among the metrologists of sport. Physicists will agree that to every measurement worth conserving for the attention of Record Committees should be attached a statement of its probable error; without such a statement there will always be the danger of proclaiming a new record on the basis of a new performance that is apparently, though not really, better than the old. If the corrections of Fig. 2 exceed the probable error to be claimed for a measurement, then those corrections must be applied.

The aim of the American Athletic Union in these matters is hard to determine. Watches must be "examined," "regulated" and "tested" by a reputable jeweler or watchmaker, but one finds no definition of what constitutes an acceptable job of regulation. Distances must be measured with "a steel tape." The Inspector of

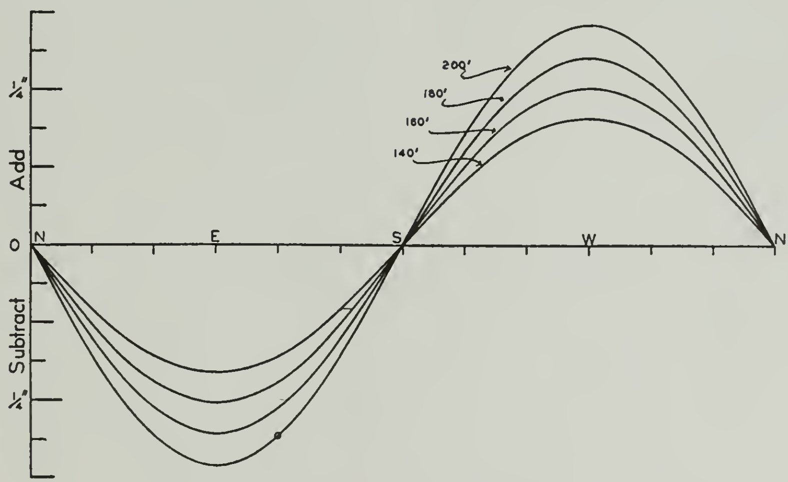


FIG. 2. Curves for rendering throws in various directions comparable. The assumed latitude is 30°, either north or south, and the assumed angle of elevation is 45°. Since the range has a maximum for about this angle of elevation, the curves also apply well to angles several degrees on either side. The curves show, for example (circled point), that a missile thrown 200 ft in a direction 30° south of east should have $\frac{5}{16}$ in. subtracted from its range in order to bring it into fair comparison with unadjusted northward or southward throws or with throws in any other direction which have been adjusted by reference to curves of this type appropriately constructed for their respective latitudes.

Implements must find the weights of the implements "correct." Such ideals of perfection are not realistic, and the only alternative is to recognize the existence of error and state its magnitude. The minimum permissible weight for each implement is prescribed both in pounds and in kilograms by AAU rules, but in no instance are the prescriptions exactly equivalent. A discus thrower whose implement just satisfies the metric specification will use a discus 4 gm, or $\frac{1}{5}$ percent, lighter than that of a competitor whose discus just passes as judged by an inspector using perfect scales calibrated in British units. Those 4 gm will give the former athlete two or three extra inches of distance, an advantage that might be decisive.

Similar comments could be made about the rules of competition of the ICAAAA, where one reads that the javelin throw is measured from the point at which the point of the javelin first strikes the ground. This is a mark that cannot in general be determined to the often implied $\frac{1}{8}$ in. since it is obliterated by the subsequent penetration of the implement. Any javelin throw as correctly measured by ICAAAA rules will show a greater distance than if measured by AAU rules, but few field judges know this nor could they do much about it if they did. It is probable that the rules do not say what was meant in these cases. It is interesting that whereas the hammer, shot and discus must be thrown upon a level surface, there is no such requirement in the case of the javelin.

Any serious attempt to put the measurements of sport upon a scientific basis would be met with vast inertia if not positive hostility. The training of athletes is still very largely an art, and there is no reason to suppose that those who are at present practicing this art with success will be predisposed to changes involving ways of thought which, however commonplace in other disciplines, are novel in athletic competition. One eminent track and field coach, a producer of national, Olympic and world champions, told the writer that he had no interest in hairsplitting; that leveling the ground accurately would be too much trouble; that common sense is better than a wind gage for estimating the effect of wind conditions on sprinters; that a man can't put the shot by theory—it's all in the feeling; that the exact angle of elevation is unimportant as long as he gets it in the groove.

A few years ago, the writer published some criticisms along the lines of the present article and sent reprints to each of the several hundred National Committeemen of the AAU. One acknowledgment was received, but no reactions to the subject matter. In a sense, this indifference was only just recompense for the writer's habit of ignoring communications from nonphysicists proposing novel theories of the atom, or otherwise instructing the physicist as to the foundations of his science.

There probably exists a general feeling that part of the charm of sport resides in accident and uncertainty. Any discussion of the possibility of replacing the balls-and-strikes umpire in baseball by a robot will bring out the opinion that the fallibilities of the umpire are part of the entertainment for which the public pays. An optical instrument for determining from the sidelines whether or not a football has been advanced to first down was tried out in California a few years ago. It was technically successful, but a popular failure. The crowd was suspicious of a measurement that it did not understand and could not watch; the players begrudged the elimination of the breather which a chain measurement affords; and even the linemen protested the loss of their dramatic moment.

Though entertained by such attitudes, the physicist will hardly be able to dismiss a feeling that in any field of popular importance or interest, it is improper to keep up the appearances of accurate and comparable measurement without doing what might be done to gain the reality. In the matter of athletic records, he and very few others know what to do about it.[3]

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[3] The author will be pleased to furnish reprints of this article to readers who would find interest in bringing it to the attention of athletic authorities.

Observation of nature by Renaissance artists and craftsmen was a precursor of the new scientific outlook. This in turn accelerated technology, leading to the industrial revolution.

15 The Scientific Revolution

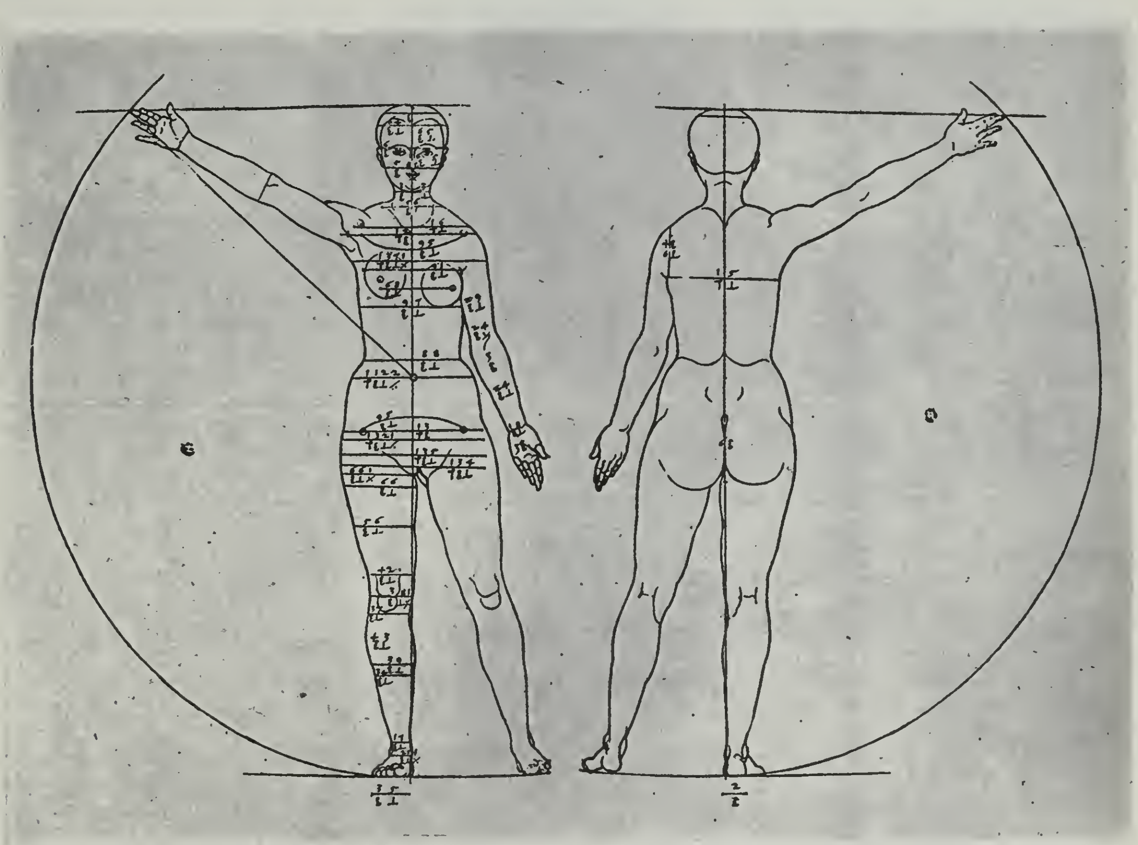
Herbert Butterfield

An article from *Scientific American*, 1960.

The preceding article leaves *Homo sapiens* in about 2500 B.C., after his invention of the city-state. Our story does not really get under way until some 4,000 years later. Thus, in turning to the next major revolution in man's impact on his environment, we seem to pass over almost all of recorded human history. No revolution is without its antecedents, however. Although the scientific-industrial age is a recent and original achievement of Western man, it has deep historical roots.

Western civilization is unique in its historical-mindedness as well as in its scientific character. Behind it on the one hand are the ancient Jews, whose religious literature was largely historical, who preached a God of history, and taught that history was moving to a mighty end, not merely revolving in cycles of growth and decay. On the other hand are the ancient Greeks. Their literature has provided a training in logic, a stimulus to the exercise of the critical faculties and a wonderful grounding in mathematics and the physical sciences.

In western Europe civilization had a comparatively late start. For thousands



ANATOMY, studied by Renaissance artists, was the first of the sciences to be placed on a modern footing. This drawing is from a copy of Albrecht Dürer's work *De Symmetria Partium Humanorum Corporum* in the Metropolitan Museum of Art in New York.

of years the lands at the eastern end of the Mediterranean had held the leadership in that whole section of the globe. It was in the West, furthermore, that the Roman Empire really collapsed, and was overrun by "barbarian invaders." Here much of the ancient culture was lost, and society reverted to comparatively primitive forms. In the meantime a high Byzantine civilization had its center in Constantinople, and a brilliant Arabian one in Baghdad. It would be interesting to know why Western man, though he started late, soon proved himself to be so much more dynamic than the peoples farther to the east.

In the formative period of a civilization religion plays a more important part than we today can easily understand. After the fall of the Roman Empire the comparatively primitive peoples in much of Europe were Christianized by conquest or through royal command; in the beginning it was a case of pagans merely changing the names of their gods. But in the succeeding centuries of the Middle Ages the Church deepened spiritual life and moral earnestness. It became the great educator, recovering ancient scholarship and acting as the patron of the arts. By the 13th century there had developed a lofty culture, very much under the presidency of religion, but a religion that nourished the inner life, stimulated heart-searchings and examinations of conscience and set an eternal value upon each individual soul. The Western tradition acquired a high doctrine of personality.

By the year 1500, when the Renaissance was at its height, the West had begun to take the command of world history. The expansion of Islam had been contained. The terrible Asian hordes, culminating in the Mongols and the Turks, that had overrun the eastern Mediterranean lands had been stopped in central Europe. One of the reasons first for survival and then for progress in the West was its consolidation into something like nation-states, a form of polity more firm and more closely knit than the sprawling Asiatic empires.

Yet the Renaissance belongs perhaps to the old (that is, the medieval) world rather than to the new; it was still greatly preoccupied with the recovery of the lost learning of ancient Greece and Rome. Its primary interest was not in scientific studies, but now, after something like a thousand years of effort, the West had recaptured virtually all it ever was to recover of ancient Greek scholarship and science. Only after this stage had been reached could the really original developments in the study of the physical universe begin. The Western mind was certainly becoming less other-worldly. In the later Middle Ages there was much thought about the nature of man as well as about the nature of God, so that a form of Christian humanism had already been developing. The Renaissance was essentially humanistic, stressing man as the image of God rather than as the doomed sinner, and it installed in western Europe the form of classical education that was to endure for centuries. The philosophy of the time dwelt much on the dignity of man. Our modern Western values therefore have deep historic roots.

GOTHIC CLOCK, dating from the early 16th century, was photographed at the Smithsonian Institution. Stone at bottom is the driving weight; arm at top is part of escapement. Clockworks were among earliest examples of well-ordered machines.

And the men of the Renaissance were still looking backward, knowing that the peak of civilization had been reached in remote antiquity, and then lost. It was easy for them to see the natural process of history as a process of decline.

Signs of something more modern had begun to appear, but they belong chiefly to the realm of action rather than to that of thought. Theories about the universe (about the movements of the planets, for example) had still to be taken over bodily from the great teachers of the ancient world. On the other hand, in action Western man was already proving remarkably free and adventurous: in his voyages of discovery, in the development of mining and metallurgy and in the creative work of the Renaissance artists. Under these conditions scientific thought might make little progress, but technology had been able to advance. And perhaps it was the artist rather than the writer of books who, at the Renaissance, was the precursor of the modern scientist.

The artists had emancipated themselves from clerical influence to a great degree. The Florentine painters, seeking the faithful reproduction of nature, sharpened observation and prepared the way for science. The first of the sciences to be placed on a modern footing—that of anatomy—was one which the artists cultivated and which was governed by direct observation. It was the artists who even set up the cry that one must not be satisfied to learn from the ancients or to take everything from books; one must examine nature for oneself. The artists were often the engineers, the designers of fortifications, the inventors of gadgets, they were nearer to the artisan than were the scholars, and their studios often had the features of a laboratory or workshop. It is not surprising to find among them Leonardo da Vinci—a precursor of modern science, but only a precursor, in spite of his brilliance, because the modern scientific method had not yet emerged.

Records show that in the 15th century a Byzantine scholar drew the attention of his fellow-countrymen to the technological superiority of the West. He mentioned progress in machine saws, shipbuilding, textile and glass manufacture and the production of cast iron. Three other items should be added to the list: the compass, gunpowder and the printing press. Although they might not have

originated in Christendom, they had not been handed down from classical antiquity. They came to be the first concrete evidence generally adduced to show that the moderns might even excel the ancients. Before 1500, artillery had assisted the consolidation of government on something like the scale of the nation-state. Printing was to speed up intellectual communication, making possible the wider spread of a more advanced kind of education and facilitating the rise of a lay intelligentsia.

In setting the stage for modern developments the economic situation is of fundamental importance. By this time a high degree of financial organization had been attained. The countryside might look much as it had done for a thousand years, but the Renaissance flourished primarily in the city-states of Italy, the Netherlands and southern Germany, where commerce and industry had made great advances. The forms of economic life were calculated to bring out individual enterprise; and in the cities the influence of priests declined—the lay intelligentsia now took the lead. There had existed greater cities and even an essentially urban civilization in ancient times. What was new was the form of the economic life, which, by the opportunities it gave to countless individuals, possessed dynamic potentialities.

It was a Western world already steeped in humanism that entered upon a great scientific and technological development. But if Western man decided now to take a hand in shaping his own destiny, he did it, as on so many other occasions, only because he had been goaded by problems that had reduced him to desperation. The decisive problems were not material ones, however. They were baffling riddles presented to the intellect.

The authority of ancient scholarship was shaken when it came to be realized that the great Greek physician Galen had been wrong in some of his observations, primarily in those on the heart. In the 16th century successive discoveries about the heart and the blood vessels were made in Padua, culminating a little later in William Harvey's demonstration in England of the circulation of the blood. The whole subject was now set on a right footing, so that a flood of further discoveries was bound to follow very quickly. Harvey's work was of the greatest importance, moreover, because it provided a pattern of what could be achieved by observation and methodical experiment.

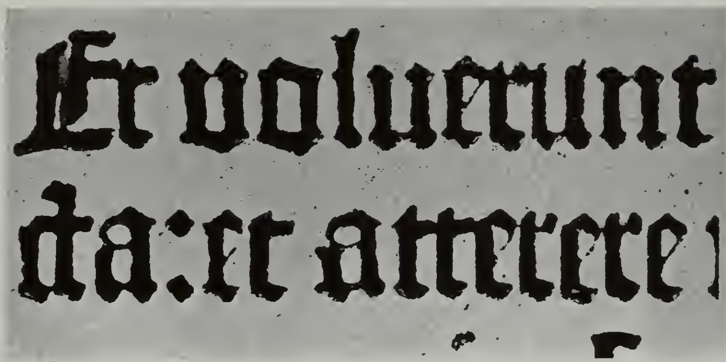
The older kind of science came to shipwreck, however, over two problems connected with motion. Aristotle, having in mind a horse drawing a cart, had imagined that an object could not be kept moving unless something was pulling or pushing it all the time. On this view it was difficult to see why projectiles stayed in motion after they had become separated from the original propulsive force. It was conjectured that a flying arrow must be pushed along by the rush of air that its previous motion had created, but this theory had been recognized to be unsatisfactory. In the 16th century, when artillery had become familiar, the student of motion naturally tended to think of the projectile first of all. Great minds had been defeated by this problem for centuries before Galileo altered the whole approach and saw motion as something that continued until something intervened to check it.

A great astronomical problem still remained, and Copernicus did not solve it alone. Accepting the recognized data, he had shown chiefly that the neatest explanation of the old facts was the hypothesis of a rotating earth. Toward the end of the century new appearances in the sky showed that the traditional astronomy was obsolete. They demonstrated that the planets, for example, instead of being fixed to crystalline spheres that kept them in their proper courses, must be floating in empty space. There was now no doubt that comets belonged to the upper regions of the sky and cut a path through what had been regarded as the hard, though transparent, spheres. It was now not easy to see how the planets were held on a regular path. Those who followed Copernicus in the view that the earth itself moved had to face the fact that the science of physics, as it then existed, could not possibly explain how the motion was produced.

In the face of such problems it began to be realized that science as a whole needed renovation. Even in the 16th century people were beginning to examine the question of method. In this case a great historic change was willed in advance and consciously attempted. Men called for a scientific revolution before the change had occurred, and before they knew exactly what the situation demanded. Francis Bacon, who tried to establish the basis for a new scientific method, even predicted the magnitude of its possible consequences —the power that man was going to ac-

COMPASS ROSE is reproduced from *The Art of Navigation*, published in France in 1666. The invention of the compass, which was not an achievement of classical antiquity, encouraged the men of the Renaissance to believe that they might come to excel the ancients.

Another momentous factor in developing the new outlook was the revival of an ancient view: that matter is composed of infinitesimally small particles. This view was now at last presented in a form that seemed consistent with Christianity (because the combinations of the particles which produced the varied world of physical things were no longer regarded as the mere product of chance), so that the atomic theory was able to acquire a wide currency. It led to a better appreciation of the intricate texture of matter, and it proved to be the source of innumerable new hypotheses. The theory seemed to open the way to a purely mechanical explanation of the universe, which should account for everything by the shape, the combination and the motion of the particles. Long before such an explanation had been achieved, men were aspiring to it. Even religious men were arguing that Creation itself would have been imperfect if God had not made a universe that was a perfectly regular machine.



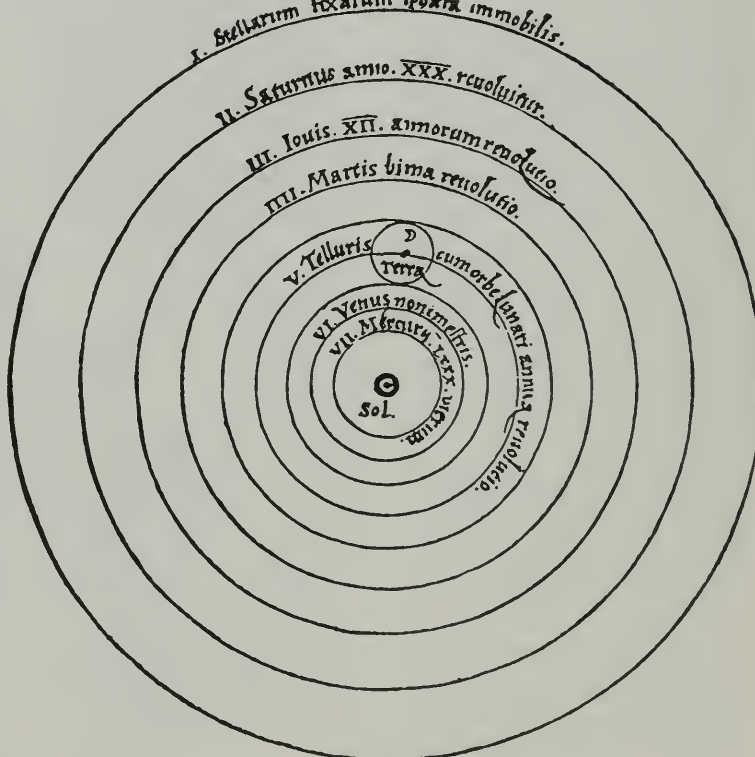
MOVABLE TYPE CAST FROM MATRICES was contribution of Johann Gutenberg to art of printing. Sample of his type, enlarged about four diameters, is from his Bible, printed about 1456. Bible in which this type appears is in Pierpont Morgan Library in New York.

quire over nature. It was realized, furthermore, that the authority of the ancient world, as well as that of the Middle Ages, was in question. The French philosopher René Descartes insisted that thinking should be started over again on a clean slate.

The impulse for a scientific revolution came from the pressure of high intellectual needs, but the tools of civilization helped to give the new movement its direction. In the later Middle Ages men had become more conscious of the existence of the machine, particularly through mechanical clocks. This may have prepared them to change the formulation of their problems. Instead of seeking the "essence" of a thing, they were now more prepared to ask, even of nature, simply: How does it work?

The student of the physical universe, like the artists before him, became more familiar with the workshop, learning manipulation from the artisan. He interested himself in problems of the practical world: artillery, pumps, the determination of longitude. Experimentation had long existed, but it now became more organized and methodical as the investigator became more conscious of what he was trying to do. In the 17th century, moreover, scientific instruments such as the telescope and the microscope came into use.

But theory mattered too. If Galileo corrected a fallacious view of motion, it was because his mind was able to change the formulation of the whole problem. At least as important as his experimentation was his mathematical attack on the problem, which illustrated the potential role of mathematics in the transformation of science.



NEW COSMOLOGY OF COPERNICUS placed a fixed sun (*Sol*) at the center of the universe. The sphere of the fixed stars (*I.*) and the spheres of the six known planets revolved around the sun. Circle inscribed around the earth (*Terra*) is the lunar sphere. This woodcut appears in Copernicus's *On the Revolution of the Celestial Spheres* (1543).

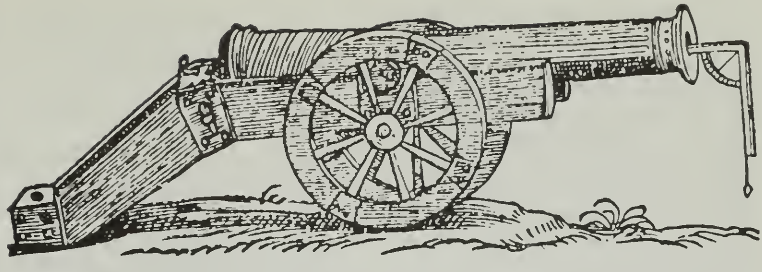
The civilization that had begun its westward shift in the later Middle Ages was moving north and west. At the Renaissance Italy still held the primacy, but with the Reformation the balance shifted more definitely to the north. By the closing decades of the 17th century economic, technological and scientific progress centered on the English Channel. The leadership now belonged to England, France and the Netherlands, the countries that had been galvanized by the commerce arising from the overseas discoveries of the 15th century. And the pace was quickening. Technique was developing apace, economic life was expanding and society was moving forward generally in an exhilarating way.

The solution of the main problems of motion, particularly the motion of the earth and the heavenly bodies, and the establishment of a new notion of scientific method, took a hundred years of effort after the crisis in the later decades of the 16th century. A great number of thinkers settled single points, or made attempts that misfired. In the period after 1660 a host of workers in Paris and London were making science fashionable and bringing the scientific revolution to its culmination. Isaac Newton's *Principia* in 1687 synthesized the results of what can now be seen to have been a century of collaborative effort, and serves to signalize a new era. Newton crowned the long endeavor to see the heavenly bodies as parts of a wonderful piece of clockwork.

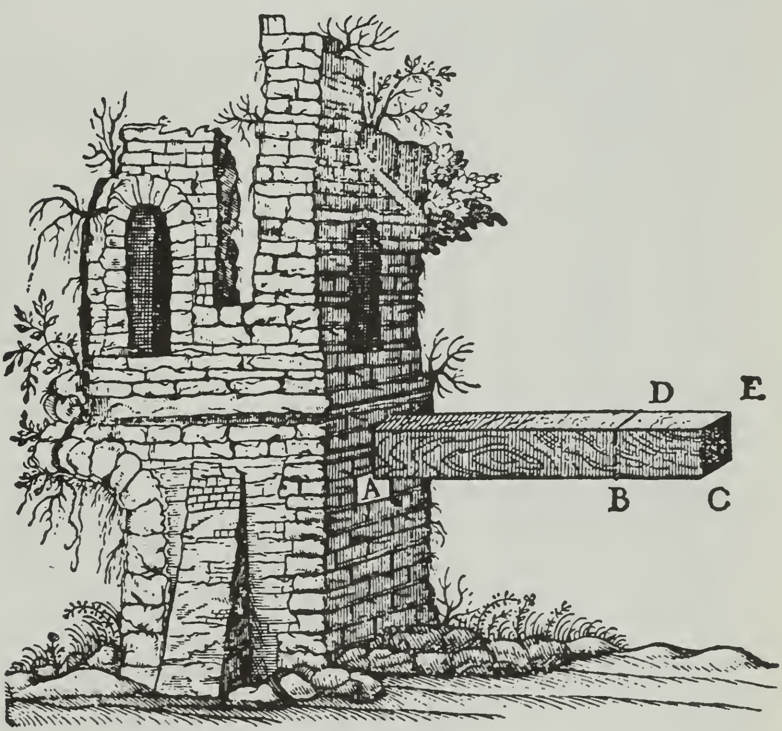
The achievements of ancient Greece in the field of science had now been unmistakably transcended and outmoded. The authority of both the ancient and the medieval worlds was overthrown, and Western man was fully persuaded that he must rely on his own resources in the future. Religion had come to a low ebb after generations of fanaticism, persecution and war; now it was in a weak position for meeting the challenge of the new thought. The end of the 18th century sees in any case the decisive moment in the secularization of European society and culture. The apostles of the new movement had long been claiming that there was a scientific method which could be adapted to all realms of inquiry, including human studies—history, politics and comparative religion, for example. The foundations of what has been called the age of reason had now been laid.

At the same time society itself was changing rapidly, and man could see it changing, see it as no longer static but dynamic. There began to emerge a different picture of the process of things in time, a picture of history as the embodiment of progress rather than of decline. The future now appeared to offer opening vistas and widening horizons. Man was coming to feel more capable of taking charge over his own destiny.

It was not merely man's tools, and not merely natural science, that had carried the story forward. The whole complex condition of society was involved, and movement was taking place on a wide front. The age of Newton sees the foundation of the Bank of England and the national debt, as well as the development of speculation that was to culminate in the South Sea Bubble. An economic order congenial to individualism meant that life was sprouting from multitudinous centers, initiatives were being taken at a thousand points and ingenuity was in constant exercise through the pressure of need or the assurance that it would have its reward. The case is illustrated in 17th-century England by the famous "projectors"—financial promoters busy devising schemes for making money. They slide easily into reformers making plans for female education or a socialistic order or a better form of government.



TRAJECTORIES OF PROJECTILES were calculated with aid of protractor device (*right*) invented by Niccolò Tartaglia, an Italian engineer and mathematician who died in 1577. Ballistics problems drew attention to the inadequacy of the Aristotelian ideas about motion.



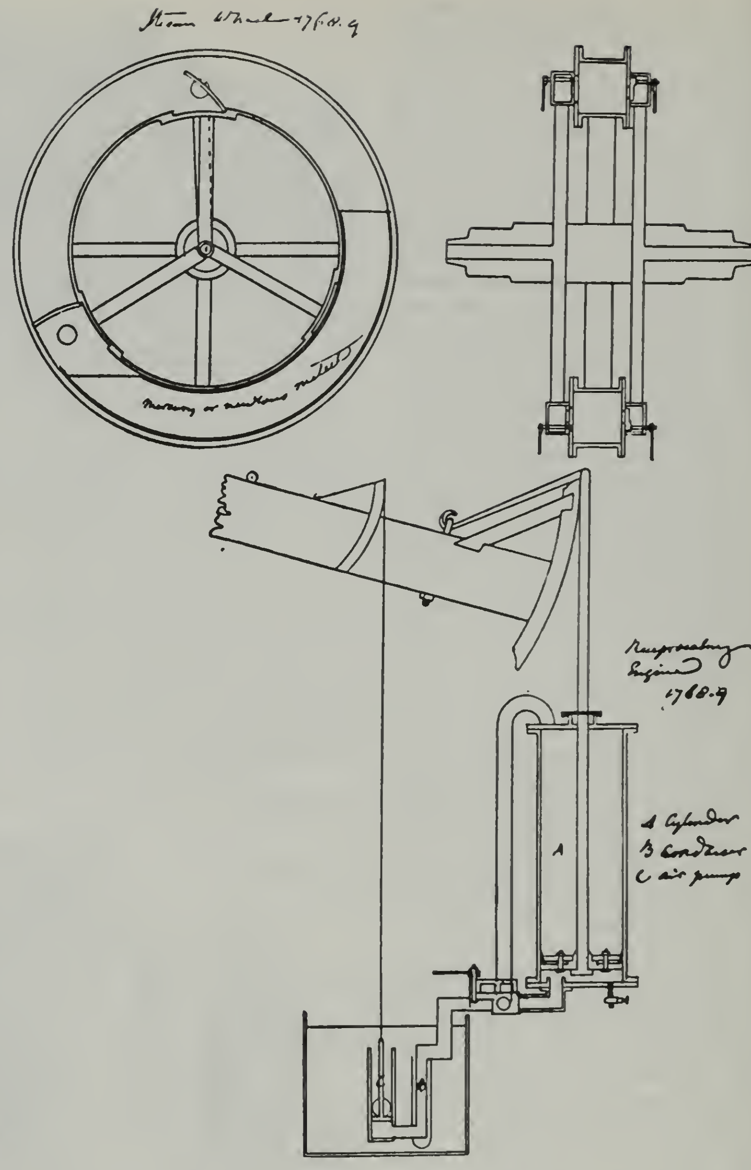
STRENGTH OF A BEAM was one of the problems in which Galileo demonstrated the power of mathematical methods in science. Illustration is taken from his *Discorsi e dimostrazioni matematiche*, in which he described the "new sciences" of mechanics and motion.

The whole of Western society was in movement, science and technology, industry and agriculture, all helping to carry one another along. But one of the operations of society—war—had probably influenced the general course of things more than is usually recognized. War above all had made it impossible for a king to "live of his own," enabling his subjects to develop constitutional machinery, to insist on terms in return for a grant of money. Because of wars, kings were allied with advanced capitalistic developments from the closing centuries of the Middle Ages. The growing demands of governments in the extreme case of war tightened up the whole development of the state and produced the intensification of the idea of the state. The Bank of England and the national debt emerge during a conflict between England and France, which almost turned into a financial war and brought finance into the very structure of government. In the 17th century armies had been mounting in size, and the need for artillery and for vast numbers of uniforms had an important effect on the size of economic enterprises.

The popularity in England of the natural sciences was paralleled to a degree by an enthusiasm for antiquarian pursuits. In the later decades of the 17th century the scientific method began to affect the development of historical study. In turn, the preoccupation with the process of things in time seems to have had an influence upon scientists themselves. Perhaps the presiding scientific achievement in the next hundred years was the application of biology, geology and allied studies to the construction of a history of the physical universe. By the end of the period this branch of science had come almost to the edge of the Darwinian theory of evolution. For the rest, if there was further scientific "revolution" in the 18th century, it was in the field of chemistry. At the beginning of the period it had not been possible to isolate a gas or even to recognize clearly that different gases existed. In the last quarter of the century Lavoisier reshaped this whole branch of science; water, which had been regarded for thousands of years as an element, was now seen to be a compound of oxygen and hydrogen.

By this time England—the nation of shopkeepers—was surprising the world with developments in the industrial field. A class of men had emerged who were agile in intellect, capable of self-help and eager for novel enterprises. They often lacked the classical education of the time, and were in a sense cut off from their cultural inheritance; and they no longer had the passion to intervene in theological controversy. Science and craftsmanship, combined with the state of the market, enabled them, however, to indulge their zeal for gadgets, mechanical improvements and inventions.

A considerable minor literature of the time gives evidence of the widespread passion for the production of technical devices, a passion encouraged sometimes by the policy of the government. Between 1760 and 1785 more patents were taken out than in the preceding 60 years; and of the estimated total of 26,000 patents for the whole century, about half were crowded into the 15 years after 1785. In 1761 the Society for the Encouragement of the Arts, Manufactures and Commerce, established a few years earlier, offered a prize for an invention that would enable six threads to be spun by a single pair of hands. A few years later Hargreave's spinning jenny and Arkwright's water frame appeared. The first steam engine had emerged at the beginning of the century, but textile factories began by using water power. The change to steam both here and in the production of iron greatly intensified the



DETAILS OF STEAM ENGINE are reproduced from James Watt's patent of 1769. The change from water to steam power in textile factories intensified the industrial revolution.

industrial revolution that was to alter the landscape so profoundly in the 19th century.

The country was able to meet the needs of a rapidly expanding population, especially as industrial development was accompanied by an agrarian revolution—the birth of something like modern farming. Possibly as a result of a change in the prevalent type of rat, England ceased to suffer from the plague that had ravaged it for centuries. Advances in public-health techniques helped reduce the death rate, especially among infants. During the 18th century the English population rose from 5.5 to nine million. And people flocked to swell the growing industrial towns, as though assured that they were fleeing from something worse to something better.

Even in 1700 most Englishmen were still engaged in occupations of a primary nature, connected with farming, fishing, mining and so on. London had perhaps half a million inhabitants, but Bristol, which came next, may have had only 20,000. Very few towns had a population exceeding 10,000. Each country town had its miller, its brewer, its tanner and so on; each village had its baker, its blacksmith and its cobbler. Many of the people who were employed in industry—in the making of textiles, for example—carried on the work in their own homes with hand looms and spinning wheels; they supplemented their income by farming.

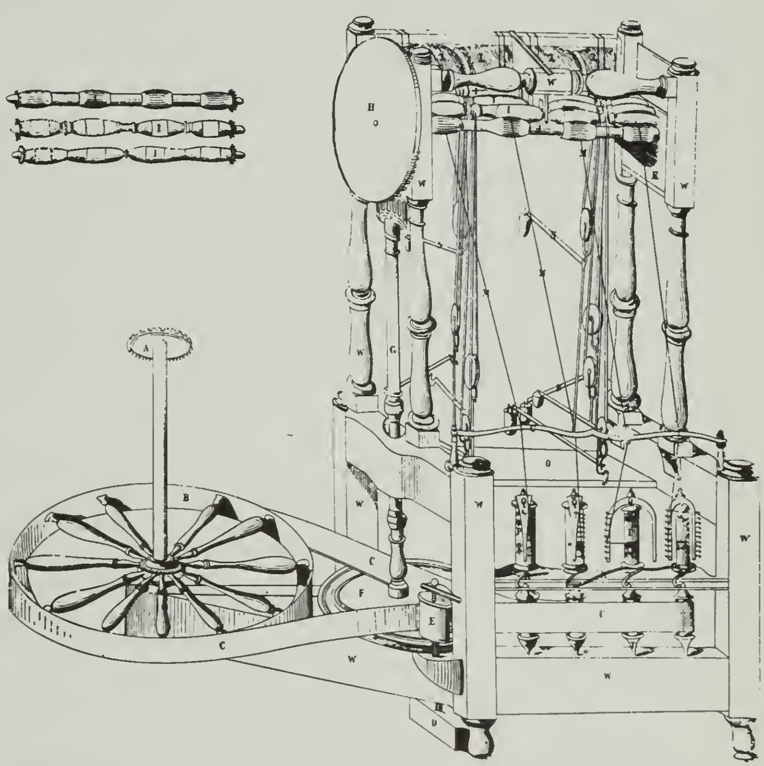
The coming of the factory system and the growth of towns represented an unprecedented transformation of life and of the human environment, besides speeding up the rate of all future change. This denser and more complicated world required more careful policing, more elaborate administration and a tremendous increase in the tasks of government. The mere growth and distribution of population, and the fresh disposition of forces that it produced within society, are fundamental factors in the history of the 19th century.

With gathering momentum came railways, the use of electricity, the internal-combustion engine and today the world of electronics and nuclear weapons. Science, so long an aid to the inventor, now seems itself to need the engineer and the industrial magnate. And all the elaborate apparatus of this technical civilization is easily communicable to every quarter of the globe. Our scientific-industrial revolution is a historical landmark for those peoples to whom Renaissance and Reformation have no relevance, since Christianity and Greek antiquity are not in their tradition. The material apparatus of our civilization is more communicable to other continents than are our more subtle and imponderable ideas.

Yet the humanism that has its roots so far back in our history has by no means lost its hold on the world. In the West, indeed, it now touches vastly wider classes of peoples than were able to read at all before the days of the industrial revolution. That revolution requires the spread of education, and at the same time provides the apparatus for it. The extraordinary speeding-up of communications and the increased mobility of life have themselves had colossal educative results. It was under the ancient order that the peasantry were sometimes felt to be like cows; John Wesley, although he held so firmly that the lowest classes were redeemable, himself described them with astonishing frequency as wild beasts. The new era has raised the stature of men, not lowered it, as some have imagined; and seems to require (or to produce) a more genuine kind of moral autonomy.

Great literature is perhaps more widely appreciated at the present day than ever in previous history. The rights and freedoms of man and the independence and self-respect of nations have never been more glorified than in our own century. And we have transmitted these ideals to other parts of the globe. The scientific-industrial revolution has operated to a great saving of life. At the same time it has provided a system which, where it has prevailed, has so far enabled the expanded population to live.

The vastness of populations and the character of the technical revolution itself have led, however, to certain dangers. The development of high-powered organization means that a colossal machine can now be put at the service of a possible dictatorship. It is not yet clear that the character of the resulting civilization will necessarily undermine the dictatorship and produce the re-establishment of what we call Western values. In this sense the elaborate nature of the system may come to undermine that wonderful individualism that gave it its start. At the same time, when nations



SPINNING FRAME, patented by Richard Arkwright in 1769, produced superior yarn. In his application the inventor said the machine would be of "great utility" to manufacturers and to the public "by employing a great number of poor people in working said machinery."

are ranged against one another, each may feel forced to go on elaborating and enlarging ever more terrible weapons, though no nation wants them or ever intends to use them. Weapons may then defeat their own ends, and man may find himself the slave of the machine.

The Western ideal of democracy is older than the scientific-industrial revolution, but it may eventually prove a necessary concomitant of that revolution, wherever the revolution may spread. At this point we simply do not know. There are certain things we cannot achieve without tools. But the tools in themselves do not necessarily determine our destiny.

The effect of the rise of physics in the age of Galileo and Newton, particularly on literature and religion, is discussed in this brief article.

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16 How the Scientific Revolution of the Seventeenth Century Affected Other Branches of Thought

Basil Willey

An article from *A Short History of Science, Origins and Results of the Scientific Revolution*, 1951.

In order to get a bird's-eye view of any century it is quite useful to imagine it as a stretch of country, or a landscape, which we are looking at from a great height, let us say from an aeroplane. If we view the seventeenth century in this way we shall be struck immediately by the great contrast between the scenery and even the climate of its earlier and that of its later years. At first we get mountain ranges, torrents, and all the picturesque interplay of alternating storm and brightness; then, further on, the land slopes down to a richly cultivated plain, broken for a while by outlying heights and spurs, but finally becoming level country, watered by broad rivers, adorned with parks and mansions, and lit up by steady sunshine. The mountains connect backwards with the central medieval Alps, and the plain leads forwards with little break into our own times. To drop the metaphor before it begins to be misleading, we may say that the seventeenth century was an age of transition, and although every century can be so described, the seventeenth deserves this label better than most, because it lies between the Middle Ages and the modern world. It witnessed one of the greatest changes which have ever taken place in men's ways of thinking about the world they live in.

I happen to be interested in literature, amongst other things, and when I turn to this century I cannot help noticing that it begins with Shakespeare and Donne, leads on to Milton, and ends with Dryden and Swift: that is to say, it begins with a literature full of passion, paradox, imagination, curiosity and complexity, and ends with one distinguished rather by clarity, precision, good sense and definiteness of statement. The end of the century is the beginning of what has been called the Age of Prose and Reason, and we may say that by then the qualities necessary for good prose had got the upper hand over those which produce the greatest kinds of poetry. But that is not

all: we find the same sort of thing going on elsewhere. Take architecture, for example; you all know the style of building called Elizabethan or Jacobean—it is quaint and fanciful, sometimes rugged in outline, and richly ornamented with carving and decoration in which Gothic and classical ingredients are often mixed up together. Well, by the end of the century this has given place to the style of Christopher Wren and the so-called Queen Anne architects, which is plain, well proportioned, severe, and purely classical without Gothic trimmings. And here there is an important point to notice: it is true that the seventeenth century begins with a blend of medieval and modern elements, and ends with the triumph of the modern; but observe that in those days to be 'modern' often meant to be 'classical', that is, to imitate the Greeks and Romans. We call the age of Dryden, Pope and Addison the 'Augustan' Age, and the men of that time really felt that they were living in an epoch like that of the Emperor Augustus—an age of enlightenment, learning and true civilisation—and congratulated themselves on having escaped from the errors and superstitions of the dark and monkish Middle Ages. To write and build and think like the ancients meant that you were reasonable beings, cultivated and urbane—that you had abandoned the shadow of the cloister for the cheerful light of the market place or the coffee house. If you were a scientist (or 'natural philosopher') you had to begin, it is true, by rejecting many ancient theories, particularly those of Aristotle, but you knew all the while that by thinking independently and taking nothing on trust you were following the ancients in spirit though not in letter.

Or let us glance briefly at two other spheres of interest: politics and religion, beginning with politics. Here again you notice that the century begins with Cavalier and Roundhead and ends with Tory and Whig—that is to say, it begins with a division arousing the deepest passions and prejudices, not to be settled without bloodshed, and ends with the mere opposition of two political parties, differing in principle of course, but socially at one, and more ready to alternate peaceably with each other. The Hanoverians succeed the Stuarts, and what more need be said? The divine right of kings is little more heard of, and the scene is set for prosaic but peaceful development. Similarly in religion, the period opens with the long and bitter struggle between Puritan and Anglican, continuing through civil war, and accompanied by fanaticism, persecution and exile, and by the multiplication of hostile sects; it ends with the Toleration Act, and with the comparatively mild dispute between the Deists and their opponents as to whether

Nature was not after all a clearer evidence of God than Scripture, and the conscience a safer guide than the creeds. In short, wherever you turn you find the same tale repeated in varying forms: the ghosts of history are being laid; darkness and tempest are yielding to the light of common day. Major issues have been settled or shelved, and men begin to think more about how to live together in concord and prosperity.

Merely to glance at this historical landscape is enough to make one seek some explanation of these changes. If the developments had conflicted with each other we might have put them down to a number of different causes, but since they all seem to be setting in one direction it is natural to suppose that they were all due to one common underlying cause. There are various ways of accounting for historical changes: some people believe, for instance, that economic causes are at the bottom of everything, and that the way men earn their living, and the way in which wealth is produced and distributed, determine how men think and write and worship. Others believe that ideas, rather than material conditions, are what control history, and that the important question to ask about any period is what men then believed to be true, what their philosophy and religion were like. There is something to be said on both sides, but we are concerned with a simpler question. We know that the greatest intellectual change in modern history was completed during the seventeenth century: was that change of such a kind as to explain all those parallel movements we have mentioned? Would it have helped or hindered that drift towards prose and reason, towards classicism, enlightenment and toleration? The great intellectual change was that known as the Scientific Revolution, and I think the answer to these questions is—Yes.

It is not for me to describe that revolution, or to discuss the great discoveries which produced it. My task is only to consider some of the effects it had upon men's thoughts, imaginations and feelings, and consequently upon their ways of expressing themselves. The discoveries—I am thinking mainly of the Copernican astronomy and the laws of motion as explored by Galileo and fully formulated by Newton—shocked men into realising that things were not as they had always seemed, and that the world they were living in was really quite different from what they had been taught to suppose. When the crystal spheres of the old world-picture were shattered, and the earth was shown to be one of many planets rolling through space, it was not everyone who greeted this revelation with enthusiasm as Giordano Bruno did. Many felt lost and confused, because

the old picture had not only seemed obviously true to common sense, but was confirmed by Scripture and by Aristotle, and hallowed by the age-long approval of the Church. What Matthew Arnold said about the situation in the nineteenth century applies also to the seventeenth: religion had attached its emotion to certain supposed facts, and now the facts were failing it. You can hear this note of loss in Donne's well-known lines:

> And new philosophy calls all in doubt;
> The element of fire is quite put out;
> The sun is lost, and th' earth, and no man's wit
> Can well direct him where to look for it.

Not only 'the element of fire', but the very distinction between heaven and earth had vanished—the distinction, I mean, between the perfect and incorruptible celestial bodies from the moon upwards, and the imperfect and corruptible terrestrial bodies below it. New stars had appeared, which showed that the heavens could change, and the telescope revealed irregularities in the moon's surface—that is, the moon was not a perfect sphere, as a celestial body should be. So Sir Thomas Browne could write:

> 'While we look for incorruption in the heavens, we find they are but like the earth;—durable in their main bodies, alterable in their parts; whereof, besides comets and new stars, perspectives (i.e. telescopes) begin to tell tales, and the spots that wander about the sun, with Phaeton's favour, would make clear conviction.'

Naturally it took a long time for these new ideas to sink in, and Milton still treats the old and the new astronomies as equally acceptable alternatives. The Copernican scheme, however, was generally accepted by the second half of the century. By that time the laws governing the motion of bodies on earth had also been discovered, and finally it was revealed by Newton that the law whereby an apple falls to the ground is the very same as that which keeps the planets in their courses. The realisation of this vast unifying idea meant a complete re-focusing of men's ideas about God, Nature and Man, and the relationships between them. The whole cosmic movement, in the heavens and on earth, must now be ascribed no longer to a divine pressure acting through the Primum Mobile, and angelic intelligences controlling the spheres, but to a gravitational pull which could be mathematically calculated. The universe turned out to be a Great Machine, made up of material parts which all moved through space and time according to the strictest rules of mechanical causation. That is to say, since every

effect in nature had a physical cause, no room or need was left for supernatural agencies, whether divine or diabolical; every phenomenon was explicable in terms of matter and motion, and could be mathematically accounted for or predicted. As Sir James Jeans has said: 'Only after much study did the great principle of causation emerge. In time it was found to dominate the whole of inanimate nature. . . . The final establishment of this law . . . was the triumph of the seventeenth century, the great century of Galileo and Newton.' It is true that mathematical physics had not yet conquered every field: even chemistry was not yet reduced to exactitude, and still less biology and psychology. But Newton said: 'Would that the rest of the phenomena of nature could be deduced by a like kind of reasoning from mechanical principles'—and he believed that they could and would.

I referred just now to some of the immediate effects of the 'New Philosophy' (as it was called); let me conclude by hinting at a few of its ultimate effects. First, it produced a distrust of all tradition, a determination to accept nothing as true merely on authority, but only after experiment and verification. You find Bacon rejecting the philosophy of the medieval Schoolmen, Browne writing a long exposure of popular errors and superstitions (such as the belief that a toad had a jewel in its head, or that an elephant had no joints in its legs), Descartes resolving to doubt everything —even his own senses—until he can come upon something clear and certain, which he finally finds in the fact of his own existence as a thinking being. Thus the chief intellectual task of the seventeenth century became the winnowing of truth from error, fact from fiction or fable. Gradually a sense of confidence, and even exhilaration, set in; the universe seemed no longer mysterious or frightening; everything in it was explicable and comprehensible. Comets and eclipses were no longer dreaded as portents of disaster; witchcraft was dismissed as an old wives' tale. This new feeling of security is expressed in Pope's epitaph on Newton:

> Nature and Nature's laws lay hid in night;
> God said, *Let Newton be!* and all was light!

How did all this affect men's religious beliefs? The effect was very different from that of Darwinism on nineteenth-century religion. In the seventeenth century it was felt that science had produced a conclusive demonstration of God, by showing the evidence of His wisdom and power in the Creation. True, God came to be thought of rather as an abstract First Cause than as the personal, ever-present God

of religion; the Great Machine implied the Great Mechanic, but after making the machine and setting it in motion God had, as it were, retired from active superintendence, and left it to run by its own laws without interference. But at a time when inherited religious sentiment was still very powerful, the idea that you could look up through Nature to Nature's God seemed to offer an escape from one of the worst legacies of the past—religious controversy and sectarian intolerance. Religion had been endangered by inner conflict; what could one believe, when the Churches were all at daggers drawn? Besides, the secular and rational temper brought in by the new science soon began to undermine the traditional foundations of belief. If nothing had ever happened which could not be explained by natural, physical causes, what about the supernatural and miraculous events recorded in the Bible? This was a disturbing thought, and even in the seventeenth century there were a few who began to doubt the literal truth of some of the biblical narratives. But it was reserved for the eighteenth century to make an open attack upon the miraculous elements in Christianity, and to compare the Old Testament Jehovah disparagingly with the 'Supreme Being' or 'First Cause' of philosophy. For the time, it was possible to feel that science was pious, because it was simply engaged in studying God's own handiwork, and because whatever it disclosed seemed a further proof of His almighty skill as designer of the universe. Addison exactly expressed this feeling when he wrote:

> The spacious firmament on high,
> With all the blue ethereal sky,
> And spangled heavens, a shining frame,
> Their great Original proclaim.
> Th' unwearied Sun from day to day
> Does his Creator's power display;
> And publishes to every land
> The work of an Almighty hand.

Science also gave direct access to God, whereas Church and creed involved you in endless uncertainties and difficulties.

However, some problems and doubts arose to disturb the prevailing optimism. If the universe was a material mechanism, how could Man be fitted into it?—Man, who had always been supposed to have a free will and an immortal soul? Could it be that those were illusions after all? Not many faced up to this, though Hobbes did say that the soul was only a function of the body, and denied the freedom of the will. What was more immediately serious, especially for poetry and religion, was the new tendency to discount

all the products of the imagination, and all spiritual insight, as false or fictitious. Everything that was real could be described by mathematical physics as matter in motion, and whatever could not be so described was either unreal or else had not yet been truly explained. Poets and priests had deceived us long enough with vain imaginings; it was now time for the scientists and philosophers to take over, and speak to us, as Sprat says the Royal Society required its members to do, in a 'naked, natural' style, bringing all things as close as possible to the 'mathematical plainness'. Poets might rave, and priests might try to mystify us, but sensible men would ignore them, preferring good sense, and sober, prosaic demonstration. It was said at the time that philosophy (which then included what we call science) had cut the throat of poetry. This does not mean that no more good poetry could then be produced: after all, Dryden and Pope were both excellent poets. But when all has been said they do lack visionary power: their merits are those of their age—sense, wit, brilliance, incisiveness and point. It is worth noticing that when the Romantic movement began a hundred years later, several of the leading poets attacked science for having killed the universe and turned man into a reasoning machine. But no such thoughts worried the men of the Augustan Age; their prevailing feeling was satisfaction at living in a world that was rational through and through, a world that had been explained favourably, explained piously, and explained by an Englishman. The modern belief in progress takes its rise at this time; formerly it had been thought that perfection lay in antiquity, and that subsequent history was one long decline. But now that Bacon, Boyle, Newton and Locke had arisen, who could deny that the ancients had been far surpassed? Man could now hope to control his environment as never before, and who could say what triumphs might not lie ahead? Even if we feel that the victory of science was then won at the expense of some of man's finer faculties, we can freely admit that it brought with it many good gifts as well —tolerance, reasonableness, release from fear and superstition—and we can pardon, and even envy, that age for its temporary self-satisfaction.

Maxwell, the developer of electromagnetic theory (Unit 4), wrote light verse. The reference in the first line of the poem is to the members of the British Association for the Advancement of Science.

17 Report on Tait's Lecture on Force, at British Association, 1876

James Clerk Maxwell

Verse written in 1876 and published in *Life of James Clerk Maxwell*, 1884.

YE British Asses, who expect to hear
Ever some new thing,
I've nothing new to tell, but what, I fear,
May be a true thing.
For Tait comes with his plummet and his line,
Quick to detect your
Old bosh new dressed in what you call a fine
Popular lecture.

Whence comes that most peculiar smattering,
Heard in our section ?
Pure nonsense, to a scientific swing
Drilled to perfection ?
That small word "Force," they make[1] a barber's block,
Ready to put on
Meanings most strange and various, fit to shock
Pupils of Newton.

Ancient and foreign ignorance they throw
Into the bargain ;
The shade of Leibnitz[2] mutters from below
Horrible jargon.
The phrases of last century in this
Linger to play tricks—
Vis Viva and *Vis Mortua* and *Vis*
Acceleratrix :—

Those long-nebbed words that to our text books still
Cling by their titles,
And from them creep, as entozoa will,
Into our vitals.
But see ! Tait writes in lucid symbols clear
One small equation ;
And Force becomes of Energy a mere
Space-variation.
Force, then, is Force, but mark you ! not a thing,
Only a Vector ;
Thy barbèd arrows now have lost their sting,
Impotent spectre !

Thy reign, O Force ! is over. Now no more
Heed we thine action ;
Repulsion leaves us where we were before,
So does attraction.

Both Action and Reaction now are gone.
Just ere they vanished,
Stress joined their hands in peace, and made them one ;
Then they were banished.
The Universe is free from pole to pole,
Free from all forces.
Rejoice ! ye stars—like blessed gods ye roll
On in your courses.

No more the arrows of the Wrangler race,
Piercing shall wound you.
Forces no more, those symbols of disgrace,
Dare to surround you :
But those whose statements baffle all attacks,
Safe by evasion,—
Whose definitions, like a nose of wax,
Suit each occasion,—

Whose unreflected rainbow far surpassed
All our inventions,
Whose very energy appears at last
Scant of dimensions :—

Are these the gods in whom ye put your trust,
Lordlings and ladies ?
The hidden[1] potency of cosmic dust
Drives them to Hades.

While you, brave Tait ! who know so well the way
Forces to scatter,
Calmly await the slow but sure decay,
Even of Matter.

This after-dinner address to the American Physical Society attempts to point up in a simplified way the amusing, as well as some of the more serious, problems which arise in connection with flight into space, including the impracticality of using the moon as a military base or of solving the population problem by colonizing the planets.

18 Fun in Space

Lee A. DuBridge

An article from *The American Journal of Physics*, 1960.

A WONDERFUL thing has happened during the past three years. A new subject has been opened up which even an old-fashioned physicist can understand. A new subject that involves no relativity corrections, no strange-particle theory—not even any Fermi statistics. Just good old-fashioned Newtonian mechanics!

Space!

All you have to do is get an object a couple of hundred miles above the earth and give it a horizontal speed of 5 or 10 miles/sec, and from that time on you can tell exactly what's going to happen to it—maybe even for a billion years—by just using Newton's laws of motion and his law of gravitation. The mathematical details get a little rough now and then, but a good IBM machine will take care of that—if you can find someone who knows how to use it. But there is nothing *in principle* that any physicist can't understand.

I personally prefer to talk about space to non-scientific audiences. In the first place, they can't check up on whether what you are saying is right or not. And, in the second place, they can't make head or tail out of what you are telling them anyway—so they just gasp with surprise and wonderment, and give you a big hand for being smart enough to say such incomprehensible things. And I never let on that all you have to do to work the whole thing out is to set the centrifugal force equal to the gravitational force and solve for the velocity. That's all there is to it! Knowing v, you can find the period of motion, of course, and that's practically all you need.

To show what I mean, let me give a simple example that I heard discussed at an IRE meeting a couple of years ago.

Imagine two spacecraft buzzing along in the same circular orbit around the earth—say 400 miles up—and one ship is 100 yards or so ahead of the other one. The fellow in the rear vehicle wants to throw a baseball or a monkey wrench or a ham sandwich, or something, to the fellow ahead of him. How does he do it?

It sounds real easy. Since the two ships are in the same orbit, they must be going at the same speed—so the man in the rear could give the baseball a good throw *forward* and the fellow ahead should catch it.

But wait! When you throw the ball out, its speed is added to the speed of the vehicle so now it is going too fast for that orbit. The centrifugal force is too great and the ball goes off on a tangent and rises to a higher orbit. But an object in a higher orbit must go slower. In fact, the faster he throws the ball, the higher it rises and the slower it goes. So our baseball pitcher stares in bewilderment as the ball rises ahead of him, then seems to stop, go back over his head, and recede slowly but surely to the rear, captured forever in a higher and slower and more elliptical orbit while the pitcher sails on his original course.

You must make a correction, of course, if you assume the ball's mass is not negligible and you take account of the conservation of momentum. Then, as the ball is pitched forward, the vehicle is slowed down—whereupon it falls into a lower orbit where, of course, it goes faster. So in this case the ball appears to rise higher and fall behind faster.

But now our ball thrower decides to try again. This time he is going to be smart. If you can't

* Text of remarks at the Banquet of the 1960 Spring Meeting of the American Physical Society, Sheraton Hall, Washington, D. C., April 27, 1960.

reach the guy ahead by throwing forward, the obvious thing to do is throw the ball to the rear. Now its speed is subtracted from that of the vehicle; hence it is going too slow for its orbit; hence it falls to a lower orbit and goes faster, passes *underneath* the rear vehicle, moves forward and passes underneath the forward vehicle, and then on into its orbit. It will be left as an exercise for the student to determine just how the baseball may be launched in order to hit the forward vehicle. One way, of course, is to first circle the earth and come back on the second lap, but there are other ways.

Now, that's all very simple Newtonian mechanics, of course. But you can see how, when you start to explain that to make an object go faster you slow it down and to make it go slower you speed it up, people begin to think you are either crazy or very smart. However, tonight I am talking to physicists and they are used to far crazier things than that—so they will have no trouble believing me at all.

So let's get on with more serious problems.

For example, last summer there appeared in a military journal an article on the use of the moon as a military base. This article is an inexhaustible source of fascinating problems for your students.

The first point made by the writer is that military men have always cherished "high ground." First a hill or a mountain, then a balloon, then an airplane, then a higher airplane, then a ballistic missile, and now—what could be more logical—the moon. Next, of course (though the author fails to mention this), comes Venus, then Mars, then Mercury, then the *sun*! Eventually, of course, we'd like to get out to Alpha Centauri (the nearest large star). But at the speeds of present space ships it would take 100 000 years or so to get to Alpha Centauri. And, who knows, the war might be over by then.

But let's stick to the moon. Our article suggests it's a real interesting possibility to hit an enemy target from the moon. The author does not mention that it would be a lot quicker, cheaper, and easier to hit it from Iowa, or Alaska, or Maine. But the moon is higher—and so is less vulnerable. Besides—here is the clincher—the velocity of escape from the moon is only 1.5 miles/sec, while the initial velocity of an ICBM is nearly 5 miles/sec. Think of all the fuel you save! Of course, there is a little matter of getting the rocket and fuel up to the moon in the first place. But that presumably will be charged to the Military Air Transport Service and so can be neglected.

Now you can easily prove that if you fired a rocket from the moon at just over 1.5 miles/sec, and did it just right, you could put it into an elliptical earth orbit which would intersect the earth's surface after a flight time of about five days. And, if you timed it just right and the earth kept spinning at just the right speed, your target might rotate into position under the point of entry just as the rocket came in. But if you made an error of a few percent in the velocity and the flight took only $4\frac{1}{2}$ days—then maybe New York would appear at the point of impact, or maybe the middle of the Pacific Ocean, or, more likely, the ellipse might miss the earth's surface entirely and the object return to its starting point. Except, the starting point, the moon—now, 10 days later—won't be there anymore! The moon will be a third of the way around its orbit!

It is, of course, very unimaginative of me not to recognize that you could shoot the rocket faster than 1.5 miles/sec and get the payload to the earth faster than five days. So you could. That takes more fuel of course—and soon you will wonder why you didn't stay home in the first place. But, the article says, you could reduce the flight time from moon to earth to a few minutes if you wished. Again, so you could. All you need to do is to accelerate to an average speed of a million miles per hour. That's 275 miles/sec. That's 55 times as fast as an earthbound ICBM, or 3000 times as much kinetic energy. So, if the ICBM takes 100 000 pounds of fuel, to launch our rocket from the moon will take 5.5 *million* pounds. And that's quite a load to get off the earth and up to the moon in the first place. In fact, you'll burn up one billion pounds of fuel just lifting it off the earth.

Well, you begin to see why space research is so much *fun*. And I think it's wonderful to have something turn up again that's fun. We always used to say that we went into physics just because it was fun. But then, with big machines and big crews and big budgets, physics research got deadly serious. I have a physicist friend who is thinking of going into biology where all he needs is a microscope and some viruses—and he can have a lot of fun. But I think space may save him for physics because that's fun too—especially if you're a theoretical physicist, as he is. As long as you don't have to go up *into* space, but can just think about it, it is a lot of fun.

There is another bundle of space problems that

can be a source of considerable amusement. Have you ever tried to explain to your wife why it is that if she were in a space capsule in an orbit around the earth she would have lost all her weight. Now the idea of losing a few pounds of weight might appeal to her, but I am sure the notion of weightlessness is something incomprehensible to most people. If you ask most laymen *why* the condition of weightlessness exists, they would tell you that since you are above the earth's atmosphere there isn't any gravity and so, of course, you must be weightless. To such people one must carefully explain that the force of gravity 200 miles above the surface of the earth is only 10% less than it is on the earth's surface. Even at 4000 miles the gravity is reduced only to one-quarter of its value on the earth's surface; and at 8000 miles, to one-ninth. Since it is obviously gravity that holds a satellite in a circular orbit, and since the earth's gravity is even strong enough out at the distance of the moon—240 000 miles—to hold the moon in its orbit, the weightlessness in an earth satellite is evidently not caused by the absence of gravity.

Then what is it caused by? Of course, if you want to be a real coward, you will choose the easy way out and simply say that in a circular orbit the force of gravity is canceled by the centrifugal force, and the condition of weightlessness results. You know very well, of course, that that isn't the proper explanation. The centrifugal force is the force that the satellite exerts on the earth and is not a force on the satellite. The force on the satellite is toward the earth and, indeed, it is the force of gravity which supplies the centripetal force which keeps the satellite in its orbit. In other words, gravity and centripetal force are in the same direction, not opposite. So, when this is pointed out by some unkind person, you get more sophisticated and say simply, "Well, in any freely falling object the condition of weightlessness exists. It would exist, for example, for passengers in a freely falling elevator." But, since not many people have been passengers in a freely falling elevator, this explanation usually falls fairly flat also. At this point I recommend that the argument be abandoned and we retreat into technical jargon by saying, "Well, it's just one of Newton's laws of motion that whenever the inertial reaction and the accelerating force are equal, no tendency toward further acceleration can exist, and hence the system behaves as though no gravitational field were present." No one can quarrel with that statement. Even if nobody understands it, it's true. And it even holds for an elliptical orbit where centrifugal force and gravity are not always equal, but weightlessness exists anyway.

By this time I suppose you will all be convinced that I am against space. However, that's not true. The Caltech Jet Propulsion Laboratory has a 50-million-dollar-a-year contract to do space research. I would not *dare* be against it!

I seriously believe that when all the popular nonsense on space is swept away, we can soberly recognize that the achievement of getting man-made vehicles into space orbits and having them transmit scientific information back to earth is one of the great triumphs in the history of technology. And, as so often happens when a new technological development occurs, new types of scientific exploration become possible.

I don't know much about the military value of space weapons. And the little I do know does not impress me. Nor do I know much about the psychological value of space ventures—how all the people in Asia and Africa think the greatest nation on earth is the one that puts up the heaviest satellite. That doesn't impress me either. But the possibilities of doing scientific experiments in space vehicles is something I can get really excited about.

Look at the very first thing that happened—the discovery of the Van Allen layers of charged particles. Think of the many exciting experiments still ahead to unravel the mysteries which that discovery opened up. And it's only the start. Now at last we can explore the earth's gravitational, magnetic, and electric fields; look down on its storm patterns; determine the nature of highly rarefied matter in the space through which the earth moves, the radiation fields present throughout space. We can now look, unimpeded, at the sun, the planets, and the stars—and a new era in astronomy is in the offing. We'll be able to examine the moon directly with instruments landed on its surface—and clear up many mysteries about the origin of the solar system. We'll discover some new mysteries too, no doubt. Mars and Venus, and eventually other planets, will soon be in the range of direct examination, too. We may actually live to see the day when we will know for sure whether the green patches on Mars are living plants or not—and, if so, whether they consist of the same type of organic molecules with which we are familiar on earth.

One of the most astonishing developments—to me at least—is that of the art of radio communi-

cation which makes it possible to transmit information over millions of miles of space. Pioneer V is being heard over 5 million miles away with only 5 w of power. Its 150-w transmitter should be heard out to 50 million miles—possibly to 100 million if we get some sensitive new receivers going in time. Clearly, objects within a distance equal to the diameter of the earth's orbit can soon be listened to—out to a quarter of a billion miles perhaps. I wonder what we can do beyond that! The inverse-square law is a pretty imposing barrier. But the ingenuity of the electronic engineer is beyond calculation. (Incidentally, as an old-time worker in the field of photoelectricity, I take especial pleasure in watching the development of the solar cell. Without it we would be in real trouble. However, when Professor Hughes and I wrote our book on *Photoelectric Phenomena*, I regarded the photovoltaic cell as such a boring subject that I was glad to let *him* write that chapter. Solar cells flying in space did not occur to us as being an imminent necessity in 1931.)

One of the most fascinating aspects of the space age is that it has given birth to a new science—*space science*. The only trouble is that no one is very clear about what space science *is*. Is it the study of the contents of space itself? If so, do we mean the space between the stars? The space between the planets? The space between the meteorites? The space between the hydrogen atoms? Or do we include everything? If we mean everything—then all the astronomers have been space scientists for 2000 years. And, if I judge correctly, many astronomers are a little disgusted with all the Johnny-come-latelys[1] who act as though they had *discovered* space—or even *invented* it. Or is space science the science you do with instruments that are *in* space? Thus, when you take pictures of the earth's clouds from a satellite, is that space science? Or is it still meteorology? When you are interested in the structure of the planet earth, you are a geologist. If you are interested in the moon, you are a *selenologist* (after Selene, the moon goddess). Is a selenologist a space scientist? Then why not a geologist too? If you are interested in Venus, then you have to look up the Greek word for Venus to find out what you are. And, since the Greek word for Venus is "Aphrodite," I still don't know what to call a Venusian geologist. Maybe "space science" isn't such a bad term after all!

All I hope is that we don't let the glamor of the term "space science" confuse us. There is a lot we can learn about the moon, for example, by just using lowly earthbound astronomical telescopes. Let's not be seduced into sending expeditions to the moon just to look for things we can see perfectly well from Palomar Mountain—or from Kitt Peak or Mt. Hamilton.

Professor Bolton and Mr. Roberts and Mr. Radhakrishnan, of the Caltech Radio Astronomy Observatory, in just a few nights observing recently found that the radio radiation from Jupiter is partially polarized and that the polarized part appears to come from a belt which is separated from the planet's disk. In other words, they have probably observed synchrotron radiation from a Van Allen belt around Jupiter. That's space science for you—and achieved in a California desert at a cost far less than the cost of even a *very small* rocket!

On the other hand, the Pioneer V package has measured the earth's magnetic field out to nearly a million miles. Preliminary analysis shows that it appears to be a pretty good dipole field out to 35 000 km, but beyond that shows small perturbations not yet analyzed. Here, clearly, is space science at its best—obtaining information available in no other way. Pioneer V is also observing charged-particle radiation far away from the earth's magnetic field—and has observed fluctuations which are correlated with disturbances on the sun. And, of course, Pioneer V is at last obtaining data on the real primary cosmic radiation. We have heard some excellent papers on space physics at this very meeting of the American Physical Society.

At last I believe the American people are beginning to realize that *these* are the real purposes of space research—to obtain scientific information. At last they are asking not just whether our satellites weigh more than the Russians', but whether they provide us with more information. We can be thankful that NASA did not yield to hysterical demands to perform useless stunts in space just to rival the Russians, but insisted on laying out a long-term program of space research. It's going to be a slow program and an expensive one. But, in the long run, solid scientific achievements will provide more national prestige than useless tricks. I believe even the Mercury man-in-space program, in spite of all the nauseating journalistic publicity

[1] At this point my secretary inserted the following note: "I suppose, if this is published, we should use 'Johnnies-come-lately,' although for oral delivery I much prefer the term you use—it has more style and zip and is more pleasing phonetically."

about the astronauts, has now been converted into a needed research program to study biological problems which must be understood by the time sending men into space becomes a really useful scientific venture.

Speaking of men in space, I am reminded of the recent television program on the population explosion in which a British economist calmly announced that rising population on earth would be no problem—we'll just ship the excess off into space! Now there is a concept to provide real merriment for your space discussions. I am told that excess population is piling up on earth at the rate of 45,000,000 people per year, or 123,000 per day. What a passenger business that's going to be! The first colony will be on the moon, I suppose. But who is going to lay the pipeline to get oxygen up to them? And water? And what about food? And space suits? With a few million people on the moon, I wonder how many space suits will get punctured every day. (A punctured space suit in a perfect vacuum is a most unpleasant accident.)

Every day! That reminds me—a day on the moon is 28 earth-days long. Sunshine for 336 hours, then darkness for 336 hours. A sizzling temperature of 220°F by day and *minus* 220° at night. In view of all the trouble, I propose instead that we build a huge floating platform all over the Pacific Ocean and put our excess population there. It would have just as much area as the moon. And, if we include the rest of the oceans, it would have as much as Mars too. And it would be a lot cheaper. And at least the people would have air to breathe!

Then we can save the moon for the people who ought to be there—physicists, chemists, biologists, geologists, and astronomers. Then, I think the moon might be an interesting place to visit!

Please forgive me for making jokes about a serious subject. My only hope is that by laughing at ourselves a little bit we may get back our sense of perspective. And a sense of perspective is important, no matter what problem we are dealing with.

In tracing the relation of science to other parts of modern life, Jacob Bronowski interviews an artist, Eduardo Paolozzi, an architect, Eero Saarinen, a physicist, Abdus Salam, and a writer, Lawrence Durrell.

19 The Vision of Our Age

J. Bronowski

A chapter from his book, *Insight-Ideas of Modern Science*, 1964.

This book began at the birth of a child, and traced its development until it enters 'the gateway to imagination and reason'. This is the stage when the child can manipulate objects in thought as well as with its hands: when it can make images of them. The child has little knowledge yet, in the ordinary sense of the word; but it has the mental equipment to learn and create knowledge. Once a child can make images, it can also reason, and build for itself a coherent picture of the world that is more than separate bundles of sense impressions.

We have just seen that when a child enters 'the gateway to imagination', it leaves all animals behind. Before it learns to make images, a young human develops in much the same way as a young animal. Children and animals alike have to learn to co-ordinate their various senses and to recognise objects. But after that, animals fall behind. They have no power of imagination. That is, they cannot carry images in the mind; and without imagery, without an inner language, they cannot manipulate ideas.

The theme of imagination runs through this book. We have examined some of the great achievements of science and seen that they are imaginative ideas. Science does not merely plod on like a surveyor, laboriously mapping a stretch of country, square mile by square mile. Of course nature must be surveyed, and very laborious that is at times; but the survey is not the end. The great moments in science come when men of imagination sit down and think about the findings—when they recreate the landscape of nature under the survey.

Science must be solidly grounded in fact and in experiment. But a blind search for experimental facts is not enough; it could never have discovered the theory of relativity. Science is a way of looking at things, an insight, a vision. And the theories of science are the underlying patterns that this way of looking at the world reveals. Many of the patterns are unexpected even at the simplest beginnings. (For example, common sense would not even have expected to find that stars and human beings are put together from the same basic building bricks of matter.) And the more unexpected the pattern, the greater the feat of imagination that is needed to see it for the first time.

What place have these imaginative ideas of science in our daily thoughts? Science and technology have transformed the physical world we live in; but have they yet had much effect on thought? Many people even dislike the ideas of science, and feel that they are abstract and mechanical. They reject science because they fear that it is in some way inhuman.

This book shows that science is as much a creation of the human imagination as art is. Science and art are not opposites; they spring from the same human impulses. In this last chapter, we shall examine their relations to one another, in the past and today. In particular, we shall see how both enter and combine into the way man in the twentieth century sees the world: the vision of our age. For this purpose, we shall include personal statements about their own work by an artist, an architect, a scientist, and a writer.

The artist is the sculptor Eduardo Paolozzi. The group of pictures show him in his studio, then one

of his sculptures being cast in the foundry, then one of his finished sculptures called *San Sebastian*—with a jet engine standing in the background—and finally another recent work.

This is what Eduardo Paolozzi had to say about his work and the world for which it is made.

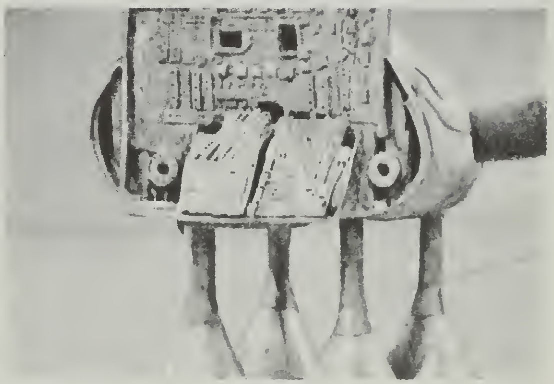
'I am a sculptor, which means that I make images. As a sculptor I was taught at the Slade the classical idea of being an artist. The best one could do would be to emulate Victorian ideals and to work in a studio executing portraits or monuments.

'But there has been a rejection now of the classical idea of tracing art out of art, which is in a way a sort of death process leading to the provincial gallery, with the atmosphere of the death-watch beetle—a gilt-edged, sure-thing idea of art.

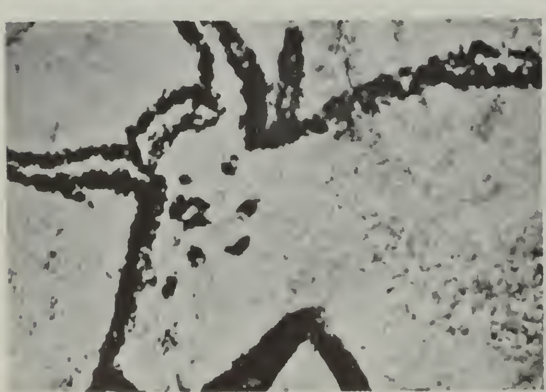
'In this century we have found a new kind of freedom—an opening up of what is possible to the artist as well as to the scientist. So I don't make copies of conventional works of art. I'm not working for Aunt Maud; I'm trying to do things which have a meaning for us living today. So I work with objects which are casual and natural today, that is, mechanisms and throwaway objects. To me they are beautiful, as my children are beautiful, though in a different way. I think they are different definitions of beauty.

'I haven't got any desire to make a sculpture of my children; but a wheel, a jet engine, a bit of a machine is beautiful, if one chooses to see it in that way. It's even more beautiful if one can improve it, by incorporating it in one's iconography. For instance, something like the jet engine is an exciting image if you're a sculptor. I think it can quite fairly sit in the mind as an art image as much as an Assyrian wine jar. I think it's a beautifully logical image, in the sense that anything in its delicate structure, with its high precision standards, has got a reason, almost in a way like human anatomy.

'My *San Sebastian* was a sort of God I made out of my own necessity: a very beautiful young man being killed by arrows, which has a great deal of symbolism in it. I think this is a good thing for young artists to identify themselves with, in a way that doing the Madonna and Child may not be a thing they can identify themselves with. It has two legs, which are decorated columns, it has a rather open, symbolical square torso, with disguised, warped, twisted, mechanic elements. Then the final element is a sort of drum with a space cut in the middle.



'What I feel about using the human diagram is that it points up in a more specific way the relationship between man and technology. There isn't any point in having a good idea in sculpture unless there is some kind of plastic or formal organisation. So I don't reproduce the jet engine, I transform it. And I use the wheel a lot in my sculpture as a symbol,



as a quickly read symbol, of the man-made object. This also refers back to my crude peasant idea of science, which is that the wheel gives the idea of man being able to get off the ground. The wheel to me is important, and the clock. I think this is very significant—I find the clock moving because I find modern science moving. I see it as a sort of heroic symbolism.

'In the last fifty years, science seems to be the outstanding leading direction, the most considerable direction that man has taken. It is trying continually to go beyond what was possible till that very moment. I think there is a possibility in what I call, crudely, higher science, a tremendous possibility of man being free. And I think it can give me a certain kind of moral strength, in the sense that art can move into a similar category of freedom. In my sculpture I am trying to speak for the way people are freeing themselves from traditional ideas. I'm a sculptor and so I put these ideas into images. If I do this well they'll be heroic images, ones that will survive and ones which other ages will recognise. Image making gives me the sense of freedom in a way that nothing else can.'

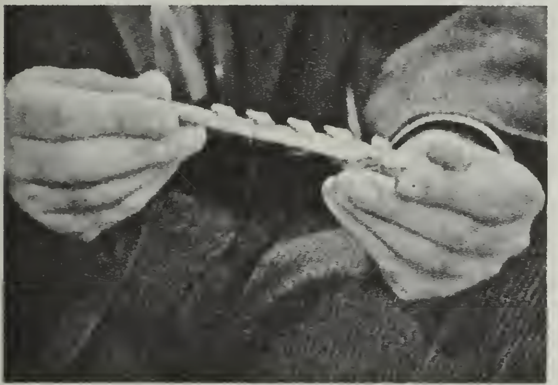
A word to which Paolozzi returns several times is 'free'. He feels that science frees man, from his conventions, from the restrictions of his environment, from his own fears and self-doubts. If this is true, then man has gained this growing freedom by imagination: in science, by imagining things that have not yet happened. Paolozzi wants to communicate the same sense of growing freedom in the images of his sculpture. He wants people to feel that they are heroic images.

Science and art are both imaginative activities, and they present two sides of the imagination. The two sides have often tried and often failed to come together, in the past and in recent time. This chapter itself, and this book, is an attempt to help bring them together. Paolozzi's work is also an attempt to bring them together, in a different language. He uses the everyday products of technology (the stamped shapes in the first picture, for example) as the raw material of his art, because they seem to him as natural and expressive in modern civilisation as the human body itself.

It is interesting to look at the two sides of the human imagination in an earlier civilisation. We have evidence for them, long even before writing was invented. These paintings, in the caves of Lascaux in southern France, are at least twenty thousand years old. They are the most famous and the finest examples of art from the Stone Age. The word 'art' is not out of place, and yet it is most unlikely that these pictures were created in the same spirit as

classical art. The caves of Lascaux were not a Stone Age art gallery that people came to visit. Art of this kind was an integral part of the civilisation of Stone Age man.

The Lascaux paintings are a product of one side of the imagination of the men who lived twenty thousand years ago. This picture shows a product of the other side of their imagination. It is a tool: a harpoon, cut from bone. It has barbs, like a modern fish hook, to stop it from being pulled out when it lodges in an animal.



The next picture shows a tool again, and of a subtler kind. It does not look as impressive as the harpoon, yet it is in fact a more far-sighted invention. For it is a tool for making tools: it is a stone graver of the sort that must have been used to cut the barbs

in the harpoon. The men who invented this were able to think beyond the immediate needs of the day—killing an animal, cutting it up, scraping its hide. When they invented a tool for making tools (today we should call that a machine-tool) they took a new step of the imagination.

What is the link between paintings on the wall of a cave, and primitive tools made of bone and flint? Separated as we are by twenty thousand years from the men who created both, we can only speculate. But we are surely right in speculating that the paint-

ings served some purpose other than mere decoration. Look at another Lascaux painting. It represents three bulls and (probably) a boar. A bull is being struck by a spear with barbs—a spear like the one that we have seen. This is plainly a hunting scene. Many of the other cave paintings show similar scenes. The painters were constantly preoccupied with hunting. This is why most authorities agree that the paintings were some kind of magic, and were intended to help the hunter to dominate the animal before the hunt started.

Unhappily, 'magic' is one of those words ('instinct' is another) that does not really explain anything. It merely says that we do not know the explanation. What kind of magic were the painters making? What did they feel they were doing for the hunters? How did they think that they were helping them to dominate the hunted animal?

Here I will give my personal view. I think that the paintings helped the men who painted them, and the men who lived in the caves with them, to conquer their fear of the hunted animal. A bull was (and is) a dangerous beast, and out in the open there would not be much time to think about him. By drawing him you become familiar with him, get used to the idea of meeting and hunting him, and imagine ways in which he can be outwitted. The close-up makes the bull familiar to you; and the familiar is never as frightening as the unknown.

It is not far-fetched here to draw an analogy with modern methods of training. Consider, for example, the training of spacemen. They have to face a frightening situation, in which what they fear is simply the unfamiliar and unknown. They will not survive if they panic; they will do the wrong thing. So a long and life-like training programme is designed to make them familiar in advance with every situation that they are likely to encounter. The spaceman's training is more than a matter of simply learning to press the right buttons. It is also a psychological preparation for the unknown.

I believe that the Stone Age cave paintings were also a psychological preparation for the unknown.

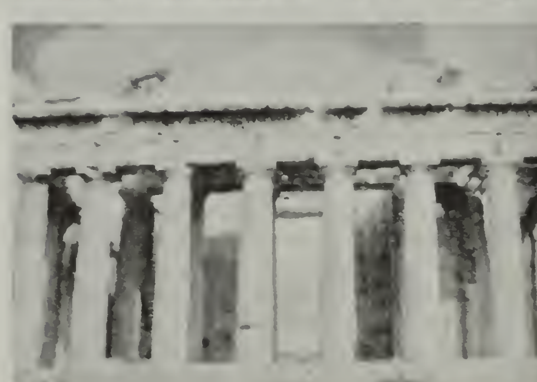
They helped the Stone Age hunters to dominate their psychological environment, just as flint and bone tools helped them to dominate their physical environment. That is the connecting link between the two. Both are tools, that is, instruments which man uses to free himself and to overcome the limitations of nature. It was Benjamin Franklin who first defined man as 'the tool-making animal'. He was right, and the tools are mental as well as physical.

We move forward now many thousand years, to a time and place where the two sides of the human imagination worked more closely together than ever before, and perhaps ever since. The pictures on the right come from Athens of the fifth century B.C. The men who built this city had suddenly burst out of the confines of the cave and come into the light of freedom. Their civilisation recognised that man's most powerful tool in the command of nature is the human mind. The Greeks named their city, and the great temple of the Parthenon in it, after the goddess of wisdom, Athene. Light and reason, logic and imagination together dominated their civilisation.

Greek architecture, for example, has a strong mathematical basis, yet it never appears stiff and mechanical. Look at the Parthenon, as perfect a creation in architecture as man has made; and it is dominated by a precise sense of numbers. Numbers had a mystical significance for the Greeks (Pythagoras made them almost into a religion) and this expressed itself in all they did.

The Parthenon has 8 columns along the front and 17 along each side. That to the Greeks was the ideal proportion. The number of columns along each side of a temple should be twice the number along the front, plus one more. No Greek architect would have built otherwise.

Numbers that are perfect squares seemed to the Greeks equally fascinating and beautiful. The Parthenon is 4 units wide and 9 units long; for 4 is the square of 2, and 9 is the square of 3—the two smallest squares. The ratio of height to width along the front of the building is also 4 to 9; and so is the ratio of the thickness of the columns to the distance between them.



Yet all this arithmetic is not a dead ritual. The Greeks found it exciting because they found it in natural objects. To them, it expressed the mystery of nature, her inner structure. Numbers were a key to the way the world is put together: this was the belief that inspired their science and their art together.

So the Parthenon is nowhere merely a set of mathematical relations. The architect is guided by the numbers, but he is never hidebound by them.

His plan begins with arithmetic, but after that the architect himself has taken command of the building, and has given it freedom, lightness and rhythm. For example, the end columns are closer to their neighbours than are the other columns; and the end columns are also a little thicker. This is to make the building compact, to make it seem to look inwards at the corners. And all the columns lean slightly inwards, in order to give the eye (and therefore the building) a feeling of upward movement and of lightness.

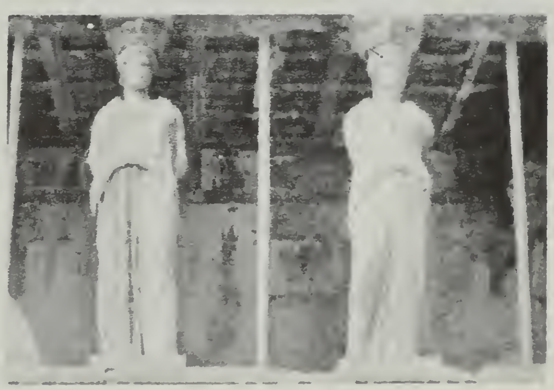
The pictures on the right are of the Erictheum. It stands close to the Parthenon, but is less famous. Perhaps that is because the Erictheum is less monumental, more slender, more delicate in its whole conception. Yet the mathematics is still there. The porch of the Erictheum, for instance, is designed on the 'golden section'. That is, the canopy has the same proportion to the base as the base has to the human figures which support the canopy. The golden section was a mathematical relation which was based on nature: on the proportions of the human body.

The human figures which support the canopy are made to seem in movement; two rest on the right foot, two on the left. Everywhere in the Erictheum there is the feeling of movement. The different levels of the building are joined together with suppleness and rhythm. This is what the Erictheum expresses in architecture: an almost musical sense of rhythm. And this reminds us that Pythagoras prided himself, rightly, on having discovered the mathematical structure of the musical scale.

The fusion of the mathematical order with the human, of reason with imagination, was the triumph of Greek civilisation. The artists accepted the mathematics, and the mathematicians did not resent the architects imposing their individuality on the mathematical framework. It was a civilisation which expressed itself in the way things were put together—buildings, ideas, society itself. Greek architecture survives to illustrate this, perhaps better than any other record.

All architecture must begin with technical efficiency. Walls have to stand up, roofs have to keep the rain out. So an architect can never be unpractical, as can a painter or a sculptor. He cannot be content with the mere look of the thing. The side of the human imagination which made the Stone Age tools cannot be left out. But a bad architect can play it down, and can take the practical for granted, as a painter takes his canvas for granted.

The strength of the best architecture today is that it does not despise the practical purposes of buildings. It does not hide the structure and function under merely elegant decoration. Structure and function in modern buildings play the same fundamental part as numbers in Greek architecture. They form



the framework on which the architect imposes his individual imagination. And he does not pretend that the framework is not there.

Our next personal statement comes from a famous architect, Eero Saarinen. He was born in Finland but built most of his great buildings in America. The pictures below show the building that he did not live to finish, the TWA Air Terminal at Idlewild Airport in New York. The lines of the building are very dramatic, and the form is consciously mathematical and aerodynamic. The question is: Is the bold, flying shape necessary, or is it a romantic artifice without a true function? I discussed this with Eero Saarinen during the building, and this is how he replied.

'To really answer your question, I would have to go a little bit back, and talk philosophically about architecture. As you know, we all, in architecture, have been working in this modern style, and certain principles have grown up within it. The basic principles are really three. There is the functional part. There is the structural part, honestly expressing the structure of the building. And the third thing is that the building must be an expression of our time. In other words, the technology of our time must be expressed in a building.

'Now those are the principles that we are all agreed on—the principles that one might have said ten years ago were the only principles. I think since that time more thought and maybe some more principles have grown up. I would say one of these additional principles, one which I believe in, is that where buildings have a truly significant purpose they should also express that purpose.'

Function and purpose were not the same thing in Saarinen's mind. The TWA Air Terminal has a clear function: to handle passengers into and out of aeroplanes. But for Saarinen, it also had a deeper purpose: from here people were to fly, and he wanted to give them the sense of freedom and adventure which flying has for earth-bound men. The vaulted shapes of the building were well-conceived as structures, but they were meant to be more: their aerodynamic and birdlike look was to express what Saarinen called the purpose—the sense of going off to fly. And the long spurs reaching out from the building show that it is not something self-contained, an end-point. They suggest entering the building and leaving it, which is of course what the passengers do.

Eero Saarinen went on:

'The last thing that I've become convinced of, and I'm not the only one, there are many others, is that once you've set the design, it must create an architectural unity. The idea of the barrel vaults making the roof of the Air Terminal building is carried through in all the details, even the furnishings.

'Basically architecture is an art, though it is halfway between an art and a science. In a way it straddles the two. I think to a large degree the motivating force in the designing of architecture comes from the arts side. If you ask, Are these curves and everything derived from mathematics? the answer is No. They are sympathetic with the forces within the vaults, which is mathematical, but there are so many choices which one has, and these really come from the aesthetic side.

'To me architecture is terribly important because it is really an expression of the whole age. After we're dead and gone, we're going to be judged by our architecture, by the cities we leave behind us, just as other times have been. What man does with architecture in his own time gives him belief in himself and in the whole period. Architecture is not just a servant of society, in a sense it's a leader of society.'

Architecture straddles art and science. That statement is true of the Greek architecture of two thousand years ago as well as of the architecture of today. In this, the Greek imagination is close to our own. The Greeks were preoccupied with the idea of structure; and we have seen in this book that the idea of

structure is also central to modern science. Like the Greeks, the modern scientist is always looking at the way things are put together, the bones beneath the skin. How often in this book have we used such phrases as 'the architecture of matter'!

For example, the Greeks invented the idea of the atom as the smallest unit of matter from which everything in the world is built. Plato thought there were five kinds of atom, and he pictured them as the five regular solids of geometry. The first four were the atoms of the four kinds of matter: earth, air, fire, and water; one of these is shown in the first picture below. The fifth was the universe itself, the unity of the other four—we still call it the quintessence; it is shown, as Plato imagined it, in the second picture.

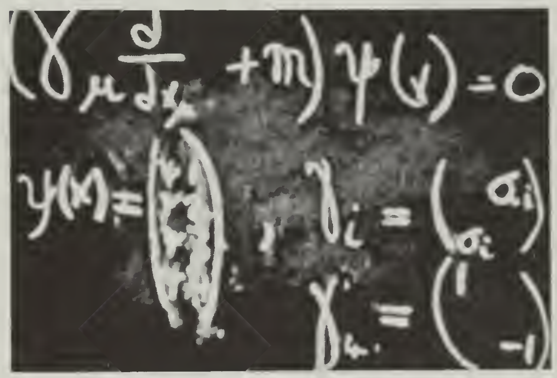
This conception is fantastic, and the atoms it pictures have no relation to the facts. And yet the fanciful pictures are a first attempt to solve, imaginatively, the same problems of structure and behaviour that the modern physicist faces. The Greek conception and the modern theories about atoms are both attempts to explain the bewildering complexity of the observable world in terms of an underlying, unifying order. Greek scientific theories are now only of historical interest. Yet before the Greeks, no one had thought about the world in this way at all. Without them, there would have been no modern science. It was the Greeks who first formulated the problems that modern science tries to answer.

Our third personal statement comes from a physicist: Professor Abdus Salam, of the Imperial College of Science in London. He describes some modern ideas about atoms. They are a long way from Plato's regular solids; yet, as Professor Salam points out, that is where they started. Here is what Salam said.

'I am a theoretical physicist, and we theoretical physicists are engaged on the following problem. We would like to understand the entire complexity of inanimate matter in terms of as few fundamental concepts as possible. This is not a new quest. It's the quest which humanity has had from the beginning of time—the Greeks were engaged on it. They conceived of all matter as being made up of fire, water, earth and air. The Arabs had their ideas about it too. Scientists have been worried about this all through the centuries. The nearest man came to solving this problem was in 1931 when, through the work done in the Cavendish Laboratory in Cambridge, we believed that all matter consisted of just two particles—electrons and protons—and all forces of nature were essentially of two kinds, the gravitational force and the electrical force.

'Now we know that this view of 1931 was erroneous. Since that time the number of particles has increased to thirty, and the number of elementary forces to four. In addition to the electrical and gravitational forces, we now believe that there are two other types of force, both nuclear—one extremely strong, and the other extremely weak. And the task we are engaged on is to try to reduce this seeming complexity to something which is simple and elementary.

'Now the type of magic which we use in order to solve our problem is first to rely on the language which we use throwing up ideas of its own. The language which we use in our subject is the language of mathematics, and the best example of the language throwing up ideas is the work of Dirac in 1928. He started with the idea that he would like to combine the theory of relativity and the theory of quantum mechanics. He proceeded to do this by writing a mathematical equation, which he solved. And to his



astonishment, and to everyone's astonishment, it was found that this equation described not only the particles—electrons and protons—which Dirac had designed the equation for, but also particles of so-called anti-matter—anti-electrons, anti-protons.

'So in one stroke Dirac had increased the number of particles to twice the number. There are the particles of matter, there are the particles of antimatter. In a sense, of course, this produces simplicity too, because when I speak of thirty particles, really

fifteen of them are particles and fifteen of them are anti-particles. The power of mathematics as a language that suggests and leads you on to something, which we in theoretical physics are very familiar with, reminds me of the association of ideas which follows when possibly a great poet is composing poetry. He has a certain rhyme, and the rhyme itself suggests the next idea, and so on. That is one type of way in which invention comes about.

'The second type of idea which we use to solve our problems is the idea of making a physical picture. A very good illustration is the work of the Japanese physicist Yukawa in 1935. Yukawa started to ponder on the problem of the attractive force between two protons, and he started with the following picture. Suppose there are two cricketers, who have a cricket ball, and they decide to exchange the ball. One throws the ball and the other catches it, perhaps. Suppose they want to go on exchanging the ball, to and fro, between them. Then the fact that they must go on exchanging the ball means that they must keep within a certain distance of each other.

'The result is the following picture. If one proton emits something which is captured by the second one, and the second one emits something which is captured by the first one, then the fact that they have to capture, emit, re-absorb constantly means that they will remain within a certain distance of each other. And someone who cannot see this intermediate object, this ball, the object we call the meson, will think that these two protons have an attractive force between them. This was Yukawa's way of explaining the attractive force between two elementary particles.

'The result of Yukawa's work was that he predicted that there do exist such particles which play the role of intermediate objects. And he predicted that such particles would have a mass about three hundred times that of electrons. Yukawa made this prediction in 1935. In 1938 these particles were discovered, and we now firmly believe that the forces of nature, all forces of nature, are transmitted by this type of exchange of intermediary particles.

'Now so far I have been talking about our methods, but what is really important are our aims. Our aim in all this is to reduce the complexity of the thirty elementary particles and the four fundamental forces into something which is simple and beautiful. And to do this what we shall most certainly need is a break from the type of ideas which I have expressed —a complete break from the past, and a new and audacious idea of the type which Einstein had at the beginning of this century. An idea of this type comes perhaps once in a century, but that is the sort of thing which will be needed before this complexity is reduced to something simple.'

The ideas put forward by Salam are vivid. But more than the specific ideas, we are interested here in his description of science itself. For him, science is the attempt to find in the complexity of nature something which is simple and beautiful. This is quite different from the usual view that science collects facts and uses them to make machines and gadgets. Salam sees science as a truly imaginative activity, with a poetic language of its own. This is an arresting point that Salam made: that the mathematics in science is a poetic language, because it spontaneously throws up new images, new ideas.

Science can learn from the language of poetry, and literature can learn from the language of science. Here we bring in our fourth contributor. He is Lawrence Durrell, who wrote the four famous books which make up *The Alexandria Quartet*. In this fourfold novel, space and time are treated in an unusual way, and Durrell began by talking about this.

'I was hunting for a form which I thought might deliver us from the serial novel, and in playing around with the notions of relativity it seemed to me that if Einstein were right some very curious by-products of his idea would emerge. For example, that truth was no longer absolute, as it was to the Victorians, but was very provisional and very much subject to the observer's view.

'And while I felt that many writers had been questing around to find a new form, I think they hadn't succeeded. I don't know of course, I've only read deeply in French. There may well be Russian or German novels which express this far better than I have.

'But they hadn't expressed what I think Einstein would call the 'discontinuity' of our existence, in the sense that we no longer live (if his reality is right) serially, historically, from youth to middle age, to death; but in every second of our lives is packed, in capsule form, a sort of summation of the whole.

That's one of the by-products of relativity that I got.

'In questing around for a means of actually presenting this in such an unfamiliar form as a novel, I borrowed a sort of analogy, perhaps falsely, from the movie camera. I'd been working with one, and it seemed to me that when the camera traverses across a field and does a pan shot, it's a historic shot in the sense that it goes from A to B to C to D. And if it starts with a fingernail and backtracks until you get a whole battlefield, that seemed to me a spatialisation. It was rooted in the time sequence that it was spatialising; it was still enlarging spatially.

'I tried to mix these two elements together, and see what would happen to ordinary human characters in what is after all a perfectly old-fashioned type of novel—an ordinary novel, only not serial. I found, somewhat to my own surprise, that I was getting a kind of stereoscopic narrative, and getting a kind of stereophonic notion of character. This excited me so much that I finished it and tried to add the dimension of time by moving the whole thing forward—you know, "read our next issue"—five years later. And there it is, ready for the critics to play with.'

Here are Lawrence Durrell's answers to some questions about his work:

Q. You said that you got from relativity the feeling that truth was provisional, or at least depended very much on the observer.

A. Well, the analogy again is the observer's position in time and space. It's so to speak the fulcrum out of which his observation grows, and in that sense it is not an absolute view, it's provisional. The subject matter is conditioned by the observer's point of view.

Q. You're really making the point that the most important thing that relativity says is that there are no absolutes?

A. I was saying, most important for me. I think that any average person who's not a mathematician would assume that that was probably the most important part of it.

Q. I want to recall another phrase that you've just used. You said of your novel that 'after all it's a perfectly old-fashioned novel'. Now I don't feel that. I feel that your novel could have been written at no time but in the twentieth century.

A. Yes, in that sense certainly. But I was trying to distinguish between the form which, I believe, if it has come off at all, is original, and the content. When I was building the form I did something new. I said to myself, this is the shape: there are three sides of space, one of time. How do I shift this notion into such an unusual domain as the novel? And at the back of my mind I wondered whether we in the novel couldn't escape our obsession with time only.

Q. Your dimensions, as it were, deepen out each character as a recession in space. You show how different he becomes when he is seen by someone else from another point.

A. Stereoscopically, you see.

Q. I want to ask you a crucial question. Do you feel that the kind of inspiration that you've drawn from the scientific idea of relativity here is valid for everyone? That we can all in some way make a culture which combines science and the arts?

A. Surely a balanced culture must do that. And I think all the big cultures of the past have never made very rigid distinctions. Also I think that the very great artists, the sort of universal men, Goethe for example, are as much scientists as artists. When Goethe wasn't writing poetry he was nourishing himself on science.

Q. We can't expect everybody to be a Goethe, so how are we going to unify what is obviously different—the sense of what the artist is doing and the sense of what the scientist is doing?

A. I think by understanding that in every generation the creative part of the population feels called upon to try and attack this mysterious riddle of what we're doing, and to give some account of themselves. We're up against a dualism, because some people have more intelligence and less emotion, and vice versa. So the sort of account they give may suddenly come out in a big poem like Dante's, or it may come out in a Newtonian concept. In other words, the palm isn't equally given in each generation. But I feel that they're linked hand in hand in this attack on what the meaning of it all is.

The meaning of it all: the meaning of the pattern of nature, and of man's place in nature. Durrell's quest is also Salam's quest, and Saarinen's, and Paolozzi's. It is the quest of every man, whether scientist or artist or man in the street.

The driving force in man is the search for freedom from the limitations which nature has imposed. Man, unlike the animals, is able to free himself. The first crude attempts were already made by Stone Age man with his tools and paintings. Now, twenty thousand years later, we are still struggling for freedom. We try to reach it by understanding the meaning of things. Our age tries to see things from the inside, and to find the structure, the architecture which underlies the surface appearance of things. We command nature by understanding her logic.

Our age has found some unexpected turns in the logic of nature. How atoms evolve, much like living species. How living things code and pass on their pattern of life, much like a machine. How the rigorous laws of nature are averaged from the million

uncertainties of atoms and individuals. How time itself is an averaging and a disordering, a steady loss of the exceptional.

How life opposes time by constantly re-creating the exceptional. And how profoundly our ideas of so safe and absolute a concept as time once seemed to be can be changed by the vision of one man, who saw and proved that time is relative.

Above all, our age has shown how these ideas, and all human ideas, are created by one human gift: imagination. We leave the animals behind because they have no language of images. Imagination is the gift by which man creates a vision of the world.

We in the twentieth century have a vision which unifies not only the physical world but the world of living things and the world of the mind. We have a much greater sense of person than any other age. We are more free than our ancestors from the limitations both of our physical and of our psychological environments.

We are persons in our own right as no-one was before us. It is not only that we can travel into space and under the oceans. Nor is it only that psychology has made us more at home with ourselves. It is a real sense of unity with nature. We see nature not as a thing but as a process, profound and beautiful; and we see it from the inside. We belong to it. This above all is what science has given us: the vision of our age.

In this chapter from her book *The Making of a Scientist,* Anne Roe, an eminent clinical psychologist, reports on her interviews with several men who became physicists. As these scientists individually describe their family backgrounds, the interests and activities of their youth, and their education, it becomes clear that there is no single pattern.

20 Becoming a Physicist

Anne Roe

An excerpt from her book *The Making of a Scientist,* 1952.

HERE ARE the stories of several of the men who became physicists. Since the theorists and experimentalists are quite unlike in some ways, I shall include both. Again it is true that some of them knew quite early that the physical sciences were a vocational possibility, and others did not hear of them in such a connection until well along in school. You can know that there is a school subject called physics, and men who teach it, and you probably will have learned that there have been fâmous men called physicists, who found out certain things about the world, but this is very different from realizing that you can make a living at finding out things in this field.

Martin was the son of a consulting engineer, who had had some college training. His mother had worked as a reporter for a while after she finished high school. He says,

"I can't remember much about grade school except the fact that I got reasonably decent grades right along and that I was fairly interested in science and mathematics. I had a friend in 7th or 8th grade who was the son of a druggist and we got a chemistry set between us and played around with it and almost blew up the house. We spent our spare time memorizing the table of elements. I never got along in languages, I couldn't see any sense in memorizing grammar. In history I read so much I had many more facts than the rest whether they were right or not. I think probably the interest in science was partly because of father. When he was home he liked to do shop work and I used to do some with him. He was rather meticulous and in some ways this was discouraging for a beginner."

Several things about this statement are very characteristic of theoretical physical scientists. All of them liked school.

Most of them preferred mathematics and science to other subjects. A number of them spoke of dabbling in chemistry, and of still being surprised that they had not blown up the house, and many of them did other sorts of things with their hands, such as the shop work mentioned by Martin. His mention of memorizing the table of elements reminds me of another of this group who became interested in mineralogy when he was a boy and who papered his room with sheets of paper on which he had copied tables and descriptions of minerals.

Martin goes on to say,

"I was rather sickly. I imagine it was more allergic than anything else, although it was not recognized at the time, and I was out sick two or three months each year. One term in high school I was only there for a month. It was always something special; my brothers and sisters always had measles and things like that but those never bothered me. I had tonsils and adenoids, hay fever, a mastoid, and appendicitis. This meant that during most of the winter months I didn't get out and I got to reading fairly early. Since I was in the 8th grade I've been in the habit of reading 4 books or more a week. I read pretty much anything. If I'm working hard in physics I like to relax by reading history or almost anything but physics. One spell in high school, when I was sick for three months, I decided I was going to go into history and I spent the time in drawing up a historical chart beginning with the Egyptians."

His frequent illnesses, and his omnivorous reading are also characteristic of this group. There were only three who had had no serious physical problems during childhood, and all of them read intensely and almost anything they could get their hands on. Two of them remarked that they thought they got their first interest in science from reading science fiction. Reading, of course, is not a very social occupation, and the physicists, like the biologists, rather tended to be quite shy. Martin, however, is unlike the others in that he got over this rather suddenly, although not very early.

"I did very little going out in high school. Mother was very worried about it. I felt very shy. I started in my junior year in college and all of a sudden found it interesting and easy and rather overdid it for a while. Let's see if I can remember how it happened. I just happened to get in with a group of fellows and girls who were interested in artistic things. I

started going to the symphony concerts at that time and we got in the habit of going Saturdays to Little Italy and sitting around and drinking wine and talking. Since that time it's been a thing I could turn on or off at will. There were a number of periods before my marriage that I did a lot of running around and other times I'd be too interested in something else. I've always been self-conscious at social functions and never cared very much for them. With a few people it's different."

In high school one of the teachers had great influence on him, and this experience oriented him towards science at the same time that out of school experiences convinced him that he did not want to be a business man. Not all of these men had occasion to spend any time in commercial activities, but quite a few of them did, usually in the course of making enough money to go to school. None of them liked business except one of the biologists who found it of interest but was glad to go back to science. The extreme competitiveness, the indifference to fact, the difficulty of doing things personally, all were distasteful to them.

"The first few years in high school I don't remember anything special about, except that I managed to get fairly decent grades in mathematics. I took physics and didn't like it. I had taken chemistry before I got there, but there was an extra course that sounded interesting so I took it and it turned out there were only four students in the course and a very interesting teacher. He sort of took personal charge and let us do pretty much what we wanted except that he was extremely insistent that we take care and do a good job. We worked through all of analytical chemistry there and I got a feeling for looking for small traces of elements, etc. This convinced me that I wanted to be a chemist. A little earlier I had gotten a job with the phone company which was with a fellow studying to be a chemist. I read Slosson, *Creative Chemistry*. This was the romantic thing to be. I think that teacher had more individual influence on me than any other."

Some firm, apparently interested in increasing the supply of chemists, had sent *Creative Chemistry* around to a number of high schools, and it seems to have been a very successful promotion. At least several others of my subjects mentioned having been influenced by it.

"When I was still in high school I took a job one summer at a Yacht Club. It was a navy camp and one of the instructors had been a radio operator. He got me interested in radio and we played around a certain amount. That winter he and two other radio amateurs decided to open a small radio equipment store in town and they asked me to go in. Perhaps they thought father might help. Dad did put up some money and we opened a small store and for a while I spent part time there. When the craze hit in 1922 or 1923 the place was about swamped, it was the only store in town. What was made on the store pretty much paid my way through college. While this episode was interesting I was pretty sure I didn't want to go into business. You always got essentially people fighting you. During part of this time in addition to working at the store I had been a part-time radio writer for one of the papers. While that was interesting, too, it didn't appeal as a life work either. By then I was convinced I wanted to go on in academic work.

"College was actually pretty much taken for granted. My mother was convinced from the beginning that all her children were going to college. I just went to college expecting to be a chemist. I had no very special idea about it. Two things happened in my freshman year. I took the college chemistry course plus the lab course. The lab course threw me for a complete loss. I think it was taught by a poor teacher who was careless of the reagents and they weren't pure. I got traces of everything and reported it. I didn't like the way the course was taught because I was told everything I was supposed to do and it soured me on chemistry.

"I got acquainted with a young man who had just come there as an astronomer and was teaching mathematics. He was perhaps the most inspiring teacher I had. He let you go if you wanted to go. I needed some money so I helped arrange the library and so I had a chance to look over the mathematics books. At the end of the year I decided the devil with chemistry, I'm going into physics.

"At that time the college had a course in physics which was not popular. My class had three students and this gave us personal attention. I thought of going on with it. My father was very dubious about it. He wasn't sure that physics was a thing you could get along with but he didn't push it very hard. He talked to me about it once and said, 'You will have to go on

in university work and won't make any money.' I said I knew that and he said 'If you realize it, that's all right.' There was nothing special about the course except at the end of that year a prize examination was given. At that time physics was taught practically everywhere without the use of calculus and still is in many places. We didn't get calculus until our sophomore year in mathematics and I still can remember the annoyance and the feeling of being cheated out of an extra year or so of activity by not having had it earlier. At any rate the physics course was given with the calculus but didn't use it. So about the middle of the second term I got disgusted and decided I wanted to learn physics the right way and asked the teacher for a text. He smiled and gave me one and I studied that so when the exam came along I gave it all in calculus and got the prize. This confirmed me, of course, and the next two years were extremely pleasant. I divided my time pretty much between astronomy and physics. There were just three of us and we'd go to the professor and say we had finished up this and what should we do next and he would say, 'What do you want to do?' So we'd tell him and he would give us manuals and get the old apparatus out and usually it would have to be cleaned and fixed up, and he would tell us to work it up and we would have a fine time.

"My teacher felt I should go on to do graduate work. This was kind of a surprise to the family and a little bit of a worry because my brothers and sisters were coming along and there wasn't too much money. But I applied for scholarships at three places and took the second offer. My main danger the first year was to keep from galloping off in 24 different directions at once. I found it extremely interesting and exciting. I started work on an experimental problem, but then I would get an idea for a theoretical paper and work on that for a while, and then go back to the other.

"I think my teacher in high school had given me a few nudges in the direction of research. Both the professors at college with whom I was in close personal contact and saw daily were active in research themselves and I just soaked that stuff up. I find it hard to think back to the time when the idea of research and just spending all the time I had available on trying to understand anything wasn't just there."

The story of George, who became an experimental physicist is quite a different one, but it is fairly characteristic of the experimentalists. He did some manual things as farm boys do, but was not particularly interested, and he did not have radio sets and gadgets of one sort or another. Farm boys didn't then. Nor did he do any particular amount of reading. So far as he knows none of his family had gone to college before him, although some have gone since; his father had had about a 6th grade education and his mother one year of high school. He started out in the usual 7 months country school, near home, but his going on was unusual. He says,

"My father and mother were rather an exception in the community which can be pointed out in this way. We lived out in the country about 7 or 8 miles from a high school. The country school to which we went was very close but when I finished seventh grade the school was having its usual ups and downs and the high school was no good. So my father and mother decided to send me to another school and it required boarding me away from home, and that was quite the talk of the area, that they would waste money boarding me.

"My recreations were the usual ones, physical activities. Whereas most parents in that neighborhood believed that children when not in school should work along with the hired help, both father and mother adopted the attitude that they expected me to do a certain amount of work but didn't care when I did it. They would lay out a certain amount per day and if I wanted to get up and work hard and be through with it that was up to me. That was always criticized because I was always enticing the other boys away when they were supposed to be at work. I earned the title of being one of the laziest boys. Father required only that I do my work and do it well. He did this with the other help as far as possible, too, like piece work. From that I learned how to make time on manual things and at the same time to do as well as required. But we had no tools and I did no carpentering. Up until I went to graduate school I never knew I had any ability in that respect at all. I didn't do a great deal of reading. In those days the books that were available were novels and I wasn't particularly interested.

"I think I wanted to go to high school. At least I was perfectly willing to go. It came rather suddenly. I don't think

very much was said about it until possibly a few days before I went. I suspect my mother had more to do with it, she had thought it out very well, but I don't think she said much even to father. His reaction was that as long as I did well he'd help me go to school. If I failed I could come home and work. He always thought farming too hard for anyone and that anyone who had intelligence would get off it. The first year or so was pretty rugged. It was difficult to find a satisfactory place to stay. We had one little course in physics in high school, not a lab course, and the usual mathematics. I think I was probably the top of the class in that.

"There was an incident there that has always been amusing to me. The only time I had any trouble in school was with the physics teacher. About the middle of the year she was showing how the water level in the boilers was determined. She left the gauge open and I said all the water would go out. The argument got hotter and hotter and finally I volunteered to show her, at which time I got thrown out of class. There again it was what father always said, you have to think things out for yourself."

This is the sort of incident that can happen when a teacher (or parent) is so insecure as to be unable to tolerate the suggestion that she might be mistaken, or might lack some particular piece of knowledge.

The experimentalists are like the theorists in their early preference for mathematics and science classes, and their disinterest in languages, and difficulty with them is somewhat greater than that shown by the theorists. Very few of the experimentalists were avid readers. The teachers at George's school were all college graduates, and the principal talked a good deal about going on to college. George was early determined to go. He liked school work, he did not like farming, and he had some idea of going into medicine. He tells how he happened to think of this.

"I started out for medicine. Along about the time I was 14, there was a young doctor came to the community and he boarded in my home. I used to drive a car for him and I got rather interested. My real interest got started from an incident one afternoon when a colored child had gotten badly burnt. Neither parent could hold the child and a neighbor couldn't do it either so he came out to the car and asked me if I thought I could hold the child and give it ether. It was

badly burned. Apparently I succeeded because that night he told my mother she had a young surgeon in the family. Maybe that started it, but when I went to college I intended to go into medicine.

"I went to the nearest college. The medicine idea shifted gradually. Two things happened, I think, that caused a shift. One was that by pure accident, in the first year mathematics course I was lucky to be in the section of an exceedingly good teacher. I always liked to be in the back of the room if I could. It seemed that during the first week this professor would start asking questions and begin at the front end, and by the time it came back to me I would have been able to get the answer, from the book or by working it out. Then he began another trick, if he didn't get the answer on the first three or four he would say, 'How about my old standby?' and call on me so I felt I had to know it. From that he began to take quite an interest in my work and before the year was out began talking about my working up the second year for myself during the summer. So I promised I'd try and he said he'd give me an examination in the fall and then I could go into the third year which he taught. I never have known if I passed it or if he let me by, but I went on with him. He wanted me to specialize in mathematics, and along with that there happened another incident.

"I had become engaged to my wife and she wasn't keen about being a doctor's wife and undoubtedly that had an influence on me. She wanted her husband at home a reasonable amount of the time. As it turned out, especially during the war, that isn't just what she got. So I gradually drifted in the direction of mathematics. The second summer I worked up some other courses and at the end of the third year had completed four years of mathematics. Along with it I took one course in physics but I wasn't particularly interested, and I had one year of chemistry. The last year I found all I lacked for a B.A. instead of a B.S. which wasn't considered as good a degree, was a year of Greek so I took that. It was a kind of training that to my mind is lacking today. I even wound up with the highest grade in the class.

"The idea of going on to graduate school came from this math professor. When I started I only intended to go through for an M.A. I didn't see my way clear further. This professor

helped me to get a fellowship and that plus my father plus my wife's working made it possible for me to go. I started out intending to spend a year and a half and get an M.A. and go out teaching in mathematics.

"Then again one of these things happened. The first summer I took two courses in mathematics and for some strange reason I was assigned a course in physics. The two courses in mathematics were taught by two foreigners and they were the two most discouraging courses I've ever had in my life. One in particular was taught by a famous English mathematician and he was teaching completely over our heads. I thought it was my own dumbness. I worked as hard as I ever worked in my life and accomplished as little. A few days before the exam I mentioned it to one of the other students and he was feeling the same way. So the next class he had the nerve to go in before the teacher came in and he went up front and asked and pretty soon he discovered most of us were in the same boat so when the professor came in we stopped him and told him this. He asked around the class and they mostly said the same. He had assumed we had had two years of mathematics that we hadn't had and so he gave an exam I could have passed in high school. I was thoroughly disgusted with mathematics. The only course that was half decent was the physics course but I wasn't prepared for that.

"At the end of the summer I thought I wouldn't go on with graduate school and I decided to go down town and get a job. If I still felt the same way I'd just continue working instead of going back next term. I got a job as a salesman. That was another lucky stroke. I went down and started putting the same effort into that. I began selling boys' shirts and I'd never bought a shirt in my life, mother always did. So I went to the library and got out three books on cloth. I read two that night and by the second day I understood a little more. I thought that if you wanted to be helpful in selling and it would be your job to learn what you were selling and it paid off as far as sales were concerned. Of course then it was said I was a sales grabber so I was told to take my turn. I said that was all right and did take my turn but I still maintained the highest sales, but it was because by then I was selecting out the good quality. I got called down for that, and they said there would be a lot of returns, but I asked them to check

it and there were hardly any. Then I had a run-in with the buyer and was transferred upstairs to sports goods and the same thing happened there. It was the same old trouble. No one ever bothered to study their stuff. At the end of the month I saw very clearly that in an industrial job you didn't get anywhere by knowing more or doing more than anyone else. By that time I was convinced that that side of the world was a pretty sorry one.

"By then I had also decided I didn't want to go on in mathematics. That one course convinced me that physics was what I wanted. I had my fellowship transferred and had a long fuss with the Dean who wanted to assign courses and I wanted to work up to them. So I started out from there and with essentially undergraduate courses.

"I liked it very much better and I found I somehow had time on my hands and very soon I wanted to try my hand in the lab. I had never had any tools in my hand. Again I had a lucky break. I went down and told the professor and said I'd like to try and I'd be glad to begin by opening boxes or anything else. He laughed and said as it happened there were a lot of boxes to open and so he put me to work. Presumably lying dormant in my fingers was an ability I didn't know I had. Within a month I challenged him that I could make an electroscope work better than he and I won. I've always wondered if he let me do it; he never would admit it but I would not expect him to.

"I found that almost anything in experimental work I had no difficulty in doing. Glass-blowing and so on just came to me overnight. I learned mainly just by doing it. Machine work was all pretty much the same way. Handling the tools just came naturally as if I had been doing it for years. So much so that when I came here and took over the shop I said I'd never ask them to do anything I couldn't do myself. At first they sometimes said they couldn't do things, but I always showed them and since then there hasn't been any question."

It is rare to find any planning ahead in the early years. Mostly the men just go from one thing to another, as occasion offers. The next story is particularly interesting from this point of view. He had an early bent to mechanical things. He went to college, largely because of his mother's dreams for him,

but even there and after he had courses in physics, it was some time before he found out about research. His story is a particularly good illustration, too, of a sort of unconsciousness about many aspects of living that is not uncommon at the college years, and not unheard of beyond them. Ernest described himself to me as an experimentalist but one of his colleagues once told me that his greatest contributions had been theoretical.

"I really can't say when I got interested in things mechanical but it's just about as early as I can remember. About 6 or so I was interested in pretty much anything electrical, the usual things that kids are interested in, autos and so on.

"Father never got even through high school and started at practically hard labor at 13 and got from that to be a star salesman. I don't know when he found time for the things he did. He was quite athletic and at that time there were amateur athletic groups and he was stroke. I never realized how good he was at the time but later I found some old papers and found that his crew was the best anywhere around. All the training was done after a day's work. Then some time later some of the books I read when I was a kid were some International Correspondence School texts on engineering which he had studied. That's a lot of work when you are working hard too. Father was a better man than I was or ever will be. Even when I was young and strong, my father was much stronger and tougher than I was always."

References to parents show marked differences in the attitudes of the sons. Ernest's respect for his father was very great, and this is generally characteristic of the physical scientists. It is less characteristic for them to have any great feeling of closeness to their fathers, or great affection, but Ernest and his father seem to have been very close.

"Father had a strong mechanical bent and I learned quite a bit from him without realizing it. From the age of ten or so I was entrusted with keeping his car serviced. By the time I was 12 there were several of us interested in radio and we made a set. I was sort of leader and I did most of the designing and construction, the others did the operating. This was a transmitting and receiving station. I was always sure I wanted to be something of the engineering sort. I had never heard the word physicist, of course, and neither had either of my parents. I had fairly large sets, Meccano and Erector,

at a rather early age. You can get a lot of action for a reasonable amount of money. The folks would buy motors for toys and when I got to be old enough to be a radio amateur I was more organized and then it was mainly a question of making up my mind what I needed. We had all kinds of complicated arrangements. For a while we formed a small company to manufacture transformers. It was sort of a joke. The power company was putting in a lot of new transformers, and so we got any amount of stuff given us by the uncle of one of the boys and then we cooked up a deal with another's uncle to dig a cellar for $20 or $30 worth of wire, and we made some transformers and sold them. I never worked so hard in my life. We sure found things out the hard way. We had considerable instruction but it was practically all of it from books and we found out how to do it the wrong way first always. It just happened there were no radio amateurs around who knew more than we did so they learned from us.

"Father never helped me make anything. On the other hand if I asked him how to do something he always knew and he had tools around which he got for his own purposes and which I appropriated so it's hard to describe. He never gave me any formal instruction but I learned a lot. Not about electricity but about mechanical things he was very, very good.

"In high school I took chemistry and physics, all there was of both, about a year of each, and then some odds and ends of surveying and such courses. I took all there was of math and some that didn't exist, i.e. the math teachers were very interested in me and awfully kind to me and gave me instruction in things that weren't really on the books and I learned some on the side myself.

"I got through high school quite young and my folks didn't think I ought to go to college quite so soon so they sent me for a year to the technical high school there, so I had perhaps better training than ordinary in that way. That was a well-run course. I spent most of my time in the machine shop.

"Going to college wasn't taken for granted. My father was the son of immigrant parents and had his first job as a blacksmith, so college tradition in the family wasn't strong. It was mother's idea. Her father was a minister and she was of a fairly well educated family. Among my boy friends none went to college. I always had had a good time in school and would

just read anything. I wouldn't say I liked all my studies but I liked anything scientific or mathematical and was all in favor of more school. Father was all for it but it was mother's idea in the first place.

"I got a scholarship and went to college intending to become an electrical engineer that being the nearest thing we knew of to what I was interested in. Then my money ran out and I went home and continued in the college there. About then I had to take sophomore courses in physics and the professor thought well of me and he said, 'Why don't you go into physics?' It seemed a lot of fun and he thought he could stir me up a job at another college and said there wasn't much difference between the physics and the electrical engineering courses and I could change back if I wanted to. I guess he must have done some considerable wrangling but he got me a job as assistant when I was a junior, and I came up here and thought that was a lot of fun.

"I was pretty young and I guess not any too noticing about some things. I didn't realize there was such a thing as research either at that time. One fine day I was downstairs and saw someone wandering down the hall with a soldering iron, something I recognized. He was a graduate student and didn't look like he knew what he was going to do so I went with him to help and spent most of my junior year working on his research and had a high old time working on it.

"This was a small place in those days. No one told me how things ran. I didn't know about any of the places where people gathered. I'd seen this fellow around the teaching labs but I'd never heard of the idea of research. I'd taken courses and I thought that teaching was what professors did. The fellow I assisted for was one of the few that did not do research and I just saw him in his teaching laboratory. I didn't have any idea of what the student I helped was trying to do. I could see he was building things that he didn't know how to do and I did so I helped him for the fun of it.

"There was an International Research Fellow here. He's a smart guy but pretty excitable and not dependable. By the time I got to be a senior it got to be recognized that I was pretty useful in the lab so they gave me to him for research associate and by that time it got time for me to graduate and I began to wonder what to do. This research Fellow was offered a job elsewhere and he could bring along anyone he

wanted so he asked me if I wouldn't like to go and I said sure. The next day I ran into the department head and told him this and he didn't say anything about it, but after a couple of weeks passed I got an offer of an instructorship here and that surprised me and I accepted. So I stayed here to get a Ph.D. I was only 20 and just had hardly grown up yet. I took chemistry too and got along well in it and had a good time. I'm sure I would have been happy as a chemist only I just had more experience of thinking mechanically that made me seem to fit into physics better.

"As it happened I worked on several problems at once, but the one I did my thesis on was a joint paper with the head, so he really suggested the problem and I just worked with him. It's a very rare student that can tell a good problem when he sees one, can start it off and carry it through. I certainly couldn't have."

Attempts to predict when things will happen, and what will be available in the future, are as fascinating as they are risky. Arthur Clarke, a science-fiction writer and scientist, has had unusual success in predicting future technical advances.

21 Chart of the Future

Arthur C. Clarke

From his book Profiles of the Future—An Inquiry into the Limits of the Possible, 1962.

THE PAST

Date	Transportation	Communication Information	Materials Manufacturing	Biology Chemistry	Physics
1800			Steam engines	Inorganic chemistry	Atomic theory
	Locomotive				
		Camera			
		Babbage calculator		Urea synthesized	
	Steamship	Telegraph	Machine tools		Spectroscope
1850					Conservation of energy
			Electricity	Organic chemistry	
		Telephone			
		Phonograph			Electromagnetism
		Office machines			Evolution
	Automobile		Diesel engine		
1900			Gasoline engine	Dyes	X-rays
	Airplane				Electron
		Vacuum tube	Mass production	Genetics	Radioactivity
				Vitamins	
			Nitrogen fixation	Plastics	
1910					
					Isotopes
		Radio			
					Quantum theory
				Chromosomes	
1920				Genes	
					Relativity
					Atomic structure
1930				Language of bees	
				Hormones	Indeterminacy
		TV			Wave mechanics
					Neutron
1940	Jet	Radar			
	Rocket				
	Helicopter				
		Tape recorders			
		Electronic computers	Magnesium from sea	Synthetics	Uranium fission
				Antibiotics	Accelerators
		Cybernetics	Atomic energy	Silicones	Radio astronomy
1950		Transistor	Automation		
	Satellite	Maser	Fusion bomb	Tranquilizers	I.G.Y.
	GEM	Laser			Parity overthrown

NOW

Date	Transportation	Communication Information	Materials Manufacturing	Biology Chemistry	Physics
1960	Spaceship	Communication satellite		Protein structure	Nucleon structure
			THE FUTURE		
	Space lab				
1970	Lunar landing	Translating machines			
	Nuclear rocket		Efficient electric storage	Cetacean languages	
1980	Planetary landings				
					Gravity waves
		Personal radio		Exobiology	
1990			Fusion power	Cyborgs	
		Artificial intelligence			
2000	Colonizing planets		"Wireless" energy		
				Time, perception enhancement	Sub-nuclear structure
		Global library	Sea mining		
2010					
	Earth probes	Telesensory devices			
			Weather control		
2020		Logical languages			Nuclear catalysts
	Interstellar probes			Control of heredity	
		Robots			
2030			Space mining		
		Contact with extra-terrestrials		Bioengineering	
2040				Intelligent animals	
			Transmutation		
2050	Gravity control			Suspended animation	
		Memory playback			
	"Space drive"		Planetary engineering		
2060		Mechanical educator			
					Space, time distortion
		Coding of artifacts		Artificial life	
2070					
			Climate control		
	Near-light speeds				
2080		Machine intelligence exceeds man's			
	Interstellar flight				
2090	Matter transmitter		Replicator		
	Meeting with extra-terrestrials	World brain		Immortality	
2100			Astronomical engineering		

Authors and Artists

PERCY WILLIAMS BRIDGMAN

P. W. Bridgman was born in Cambridge, Massachusetts in 1882, entered Harvard in 1900, received his Ph.D. in physics there in 1908, and in 1913 became Professor. He retired in 1954, and died in 1961. Bridgman's experimental work was in high-pressure physics, for which he received the Nobel Prize in 1946. He has made important contributions to philosophy of science; for example, we owe him first detailed articulation of the concept of operational definition.

JACOB BRONOWSKI

Jacob Bronowski, who received his Ph. D. from Cambridge University in 1933 is now a Fellow of the Salk Institute of Biological Studies in California. He has served as Director of General Process Development for the National Coal Board of England, as the Science Deputy to the British Chiefs of Staff, and as head of the Projects Division of UNESCO. In 1953 he was Carnegie Visiting Professor at the Massachusetts Institute of Technology.

HERBERT BUTTERFIELD

Herbert Butterfield is Professor of Modern History at the University of Cambridge. He graduated from Cambridge and was elected a Fellow of Peterhouse at the same institution in 1923. He became Master of Peterhouse in 1955 and vice chancellor of the University in 1959. His writings include books on the history of religion, international affairs, and the history of science.

ALEXANDER CALDER

Alexander Calder, the American sculptor and inventor of the mobile, was born in Pennsylvania in 1898. Intending to become an engineer, Calder entered the Stevens Institute of Technology, graduating in 1919. But by 1926 he had already published his first book (Animal Sketches) and presented his first exhibition of paintings. A visit with the Dutch artist Piet Mondrian in 1930 oriented him toward abstraction, and the next year he produced the first "stabiles," and in 1932, the first "mobiles." In these mobiles, Calder was able to incorporate motion into sculpture.

ARTHUR C. CLARKE

Arthur C. Clarke, British scientist and writer, is a Fellow of the Royal Astronomical Society. During World War II he served as technical officer in charge of the first aircraft ground-controlled approach project. He has won the Kalinga Prize, given by UNESCO for the popularization of science. The feasibility of many of the current space developments was perceived and outlined by Clarke in the 1930's. His science fiction novels include Childhoods End and The City and the Stars.

LEE DuBRIDGE

Lee DuBridge was born in Terre Haute, Indiana in 1901, and educated at Cornell College (Iowa) and the University of Wisconsin. During World War II he served as Director of the Radiation Laboratory at the Massachusetts Institute of Technology, where Radar was perfected. In 1946 he became the president of the California Institute of Technology and served in that capacity until becoming the Adviser to President Nixon on Science and Technology. His special fields of interest include biophysics, nuclear physics, and photoelectric and thermionic emission.

RICHARD PHILLIPS FEYNMAN

Richard Feynman was born in New York in 1918, and graduated from the Massachusetts Institute of Technology in 1939. He received his doctorate in theoretical physics from Princeton in 1942, and worked at Los Alamos during the Second World War. From 1945 to 1951 he taught at Cornell, and since 1951 has been Tolman Professor of Physics at the California Institute of Technology. Professor Feynman received the Albert Einstein Award in 1954, and in 1965 was named a Foreign Member of the Royal Society. In 1966 he was awarded the Nobel Prize in Physics, which he shared with Shinichero Tomonaga and Julian Schwinger, for work in quantum field theory.

JAMES BASIL GERHART

James Gerhart is Professor of Physics at the University of Washington in Seattle. Before coming to Washington, he taught at Princeton, where he received his Ph.D. in 1954. Professor Gerhart's specialty is nuclear physics.

J. B. S. HALDANE

J. B. S. Haldane was a British geneticist who served as Professor of Biometry at University College, London. He pioneered in the application of mathematics to the study of natural selection and to other aspects of evolutionary theory. His broad grounding in mathematics, physics, and biology has enabled him to make insightful contributions in many different areas.

BANESH HOFFMANN

Banesh Hoffman, born in Richmond, England in 1906, attended Oxford and Princeton. He has been a member of the Institute of Advanced Study, electrical engineer at the Federal Telephone and Radio Laboratories, researcher at King's College, London, and a consultant for Westinghouse Electric Corporation's science talent search tests. He has won the distinguished teacher award at Queens College, where he is Professor of Mathematics. During the 1966–1967 year he was on the staff of Harvard Project Physics.

GERALD HOLTON

Gerald Holton received his early education in Vienna, at Oxford, and at Wesleyan University, Connecticut. He has been at Harvard University since receiving his Ph.D. degree in physics there in 1948; he is Professor of Physics, teaching courses in physics as well as in the history of science. He was the founding editor of the quarterly Daedalus. Professor Holton's experimental research is on the properties of matter under high pressure. He is a co-director of Harvard Project Physics, the group that developed materials on which the Project Physics Course is based.

FRED HOYLE

Fred Hoyle is an English theoretical astronomer, born in Yorkshire in 1915. Now Professor of Astronomy at Cambridge University, he is perhaps best known for one of the major theories on the structure of the universe, the steady-state theory. Hoyle is well known for his scientific writing, and his success in elucidating recondite matters for the layman.

GYORGY KEPES

Gyorgy Kepes was born in 1906 in Selyp, Hungary. From 1930 to 1936 he worked in Berlin and London on film, stage, and exhibition design. In 1937 he came to the United States to head the Light and Color Department at the Institute of Design in Chicago. Since 1946 he has been Professor of Visual Design at the Massachusetts Institute of Technology. He has written The New Landscape in Art and Science, Language of Vision, and edited several books, including those in the Vision + Value series. Professor Kepes is one of the major painters; his work is included in the permanent collections of many museums.

PAUL KIRKPATRICK

Born in South Dakota, Paul Kirkpatrick received his doctorate in physics in 1923. Before reaching Stanford in 1931, he taught in China and Hawaii. At Stanford, he was named Professor of Physics in 1937, and became Professor Emeritus in 1959. Professor Kirkpatrick has served as education advisor with the U.S. Overseas Mission to the Philippines, and with the UNESCO mission to India.

JAMES CLERK MAXWELL

James Clerk Maxwell was born in Edinburgh, of a prominent Scottish family, in 1831. He graduated second in his class in mathematics at Cambridge, and was appointed to a professorship at Aberdeen in 1856. Shortly thereafter he demonstrated that Saturn's rings were composed of small particles.. Next, Maxwell considered the mechanics of gases, and helped develop the kinetic theory. Maxwell's crowning achievement was his mathematical formulation of the laws of electricity and magnetism. He showed that electricity and magnetism were related, and proposed that light was one form of electromagnetic radiation. In 1871, Maxwell was appointed first Professor of Experimental Physics at Cambridge. He died eight years later, his life cut short by cancer.

HERBERT MATTER

Herbert Matter was born in Engelberg, Switzerland, on April 25, 1907. After graduating from college, he studied painting at L'Ecole des Beaux Arts in Geneva, and under Fernand Leger in Paris. In 1936 he came to the United States to work as a free-lance photographer for Harper's Bazaar, Vogue, and others. Presently he is Professor of Photography and Graphic Design at Yale University.

RUDI HANS NUSSBAUM

Rudi Nussbaum was born in Germany in 1922, he received his Ph.D. from the University of Amsterdam in experimental physics in 1954. Since then he has served as UNESCO research fellow at the Nuclear Physics Laboratory in Liverpool, as a senior fellow at CERN in Geneva, and is now Professor of Physics at Portland State College.

GEORGE POLYA

George Polya was born in Budapest in 1887. He studied in Vienna, Göttingen, and Budapest, where he received his doctorate in mathematics in 1912. He taught in Zurich, and in this country at Brown University, Smith College, and Stanford University, where he served as Professor of Mathematics from 1946 to 1953. He is now Professor Emeritus.

JACOPO DA PONTORMO (JACOPO CARRUCCI)

Born at Pontormo, Italy, May 24, 1494, Jacopo Carrucci, later to be known as Jacopo da Pontormo, was one of the first of the Florentine Mannerists. Apprenticed to Leonardo da Vinci and later to Albertinelli and Piero di Cosimo, Pontormo broke away from the classical High Renaissance style. His altarpiece (still in the church of S. Michele Visdomini, Florence) exemplifies his intensely emotional style, in contrast to the traditional harmonically balanced style. Pontormo was buried in Florence on January 2, 1557.

ANNE ROE

Anne Roe, a psychologist and educator, born in Denver, Colorado, was educated at the University of Denver and Columbia University. From 1947 to 1951 she was the director of a psychological study of scientists, that resulted in the book The Making of a Scientist. She is the wife of biologist George Gaylord Simpson.

DUANE H. D. ROLLER

Duane H. D. Roller was educated at Columbia University, Purdue University and at Harvard University. Since 1954 Dr. Roller has been at the University of Oklahoma, where he is McCasland Professor of the History of Science.

C. L. STONG

C. L. Stong was born in 1902 in Douds, Iowa. He attended the University of Minnesota, the Armour Institute in Chicago, and the University of Michigan (Detroit). For thirty years he was an engineer with Western Electric. Mr. Stong has also been involved in movie production, and in the early 1920's he was a stunt flier. Since 1948 he has been a contributor to Scientific American, where his column, The Amateur Scientist, appears monthly.

WARREN WEAVER

Warren Weaver received his Ph.D. in mathematics and physics from the University of Wisconsin in 1921, and remained at his alma mater, becoming Professor of Mathematics and Chairman of the Department in 1928. In 1932 he was appointed Director of Natural Sciences at the Rockefeller Foundation, and in 1955 was named Vice-president. He later was associated with the Sloan-Kettering Institute, and since 1959 with the Alfred P. Sloan Foundation. He is the recipient of the Arches of Science Award given by the Pacific Science Center of Seattle "for outstanding contributions to the improved public understanding of science."

BASIL WILLEY

Basil Willey was born in 1897 and later attended Peterhouse College, Cambridge, where he read history and English. From 1946 to 1964 he served as King Edward VII Professor of English Literature at Cambridge. In 1958 he was selected as President of Pembroke College, Cambridge, and is now an Honorary Fellow. His published works include many studies in English and the history of ideas.

Holt/Rinehart/Winston